THERMAL AND ELECTRICAL CONDUCTIVITY OF METALS

1 Objectives

- T understand the thermal and electrical conductivity of metals.
- To measure the coefficient of thermal conductivity of a metal (Cu or Al).
- To measure the electrical conductivity of a metal (Cu or Al).
- To check the Wiedemann-Franz law and find out the Lorenz number.

2 Theorethical background

The great success of the Drude's model for the metals was to explain the Wiedemann-Franz law. According to this last law, the ratio between the thermal conductivity coefficient and the electrical conductivity is proportional to the temperature; and, the proportionality constant depends only on universal constants.

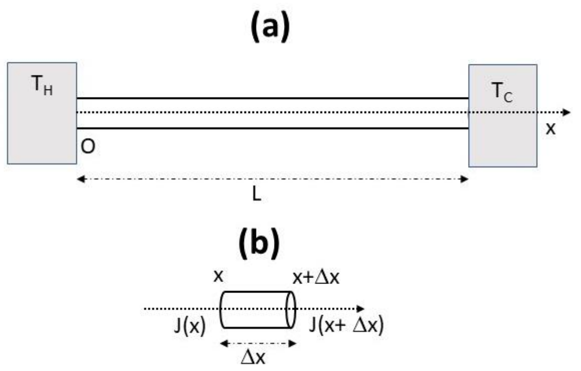
2.1 Thermal conductivity in metals

In a metal, at moderate temperatures the thermal conductivity is due to the heat transported in the degree of freedom of the electrons that form the electron gas. The thermal current follows the Fourier's law:

$$\vec{J} = -\kappa \vec{\nabla} T \quad (1)$$

Where **J** represents the flow of thermal energy per unit of time crossing a unit area perpendicular to the thermal flow, κ is the coefficient of thermal conductivity and $\vec{\nabla} T$ is the gradient of temperature in the material.

A better understanding of the thermal conductivity can be achieved if it is considered a metallic rod arranged along the x-axis, whose ends are in contact with a heat and hot reservoirs, as it is shown in the figure 1a. If the temperature of the rod is measured at a certain point, it is observed that the temperature depends both on time and on the position, so T = T(t,x). The equation that gives the temperature distribution along the rod and as a function of time, T(t,x), is very easy to determine from equation (1).



Let us consider an element of the rod Δx (see figure 1b). The heat per unit of time entering into ΔX is J(x)S and the heat per unit of time going out from Δx is J(x+Δx)S. S is the cross-section of the rod. Therefore, it is verified:

$$J(x)S - J(x+\Delta x)S = J(x)S - \left(J(x) + \frac{\partial J}{\partial x}\Delta x \right) S = -\frac{\partial J}{\partial x} S\Delta x = \frac{dQ_{\Delta x}}{dt} \quad (2)$$

The variation of temperature of the element Δx can be obtained if the heat specific of the bar c_e is considered since:

$$\frac{dQ_{\Delta x}}{dt} = \rho S \Delta x c_e \frac{\partial T}{\partial t} \quad (3)$$

$$\Rightarrow \rho c_e \frac{\partial T}{\partial t} = -\frac{\partial J}{\partial x} \quad (4)$$

If the equation (1) is taken into consideration, it is obtained the following equation that T(t,x) must accomplish:

$$\frac{\partial T}{\partial t} = \frac{\kappa}{\rho c_e} \frac{\partial^2 T}{\partial x^2} \quad (5)$$

The solution of equation (5) implies two terms: a transitory term and a stationary term. Once the system is in the stationary term, the temperature along the rod does not depend on the temperature, so

$$\frac{\partial T}{\partial t} = 0 = \frac{\kappa}{\rho c_e} \frac{\partial^2 T}{\partial x^2} \Rightarrow \frac{dT}{dx} = cte \quad (6)$$

In this case, if L is the length of the rod, the temperature at any pint of the rod is given by the expression:

$$T(x) = -\left(\frac{T_H - T_C}{L}\right) x + T_H \quad (7)$$

In the stationary regime, the density of thermal current **J** is constant. It means that the heat per unit of time deliver to the rod by the hot reservoir coincides with the heat delivers by the rod to the cold reservoir. So, if the hot per unit of time delivered buy the rod to the cold reservoir is measured it is possible to find out the coefficient of thermal conductivity of the rod by using the expression (1). In this case:

$$\frac{dQ_{meas}}{dt} = JS = -\kappa S \frac{dT}{dx} \quad (8)$$

The left term of the equation (8), $\frac{dQ_{meas}}{dt}$, is experimentally measured, and the term of the right of the equation, $\frac{dT}{dx}$ can be determined from the slope of equation (7), or just from measuring the temperature of two different points of the rod separated by a known distance. In this way, from equation (8), it is possible to obtain, κ, the coefficient of thermal conductivity.

Int his practice, in order to determine the coefficient of thermal conductivity of a material like Cu or Al, it is considered a rod. The ends of rod must be in contact with two heat reservoirs. The hot reservoir is a calorimeter with water that is boiling. In this way, we can be sure that temperature is known, 100 ºC and besides it remains constant. The cold reservoir is other calorimeter with water a 0 ºC, and for it, in the calorimeter there will be liquid water and ice at 0 ºC. The temperature in different points of the rod is measured until there are evidences that the temperature does not vary with time, only varies with the position. Then, it means that the stationary regime has been achieved. At this moment, the heat transferred from the bar to the cold calorimeter is measured. For it, the ice of the cold reservoir is taken out, and it is measured how the calorimeter temperature (cold reservoir) increases.

2.2 Electrical conductivity in metals

The electrical conductivity in a metal is described by the Ohm's law: RI = V. I is the electrical current intensity flowing through the metal, V is the potential drop and R is the electrical resistance. In case the metal is a rod of length L and cross section S (figure 1a), the resistance of the rod is

$$R = \frac{L}{\sigma A} \quad (9)$$

Where s is the electrical conductivity of the bar. So, according to the Ohm's law:

$$V = \frac{L}{\sigma A} I \quad (10)$$

On performing the experimental set-up of the Figure 2, it is possible to determine the electrical conductivity of a copper or aluminium rod.

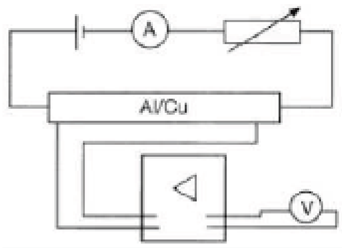


Figure 2. Experimental set-up for measuring the electrical conductivity of a rod

By using a variable resistance, it is possible to vary the intensity of the electric current flowing through the rod and if the potential difference between two points of the rod is measured, on representing V versus I, according to the expression (10) it is possible to determine the electrical conductivity of the metal.

2.3 Wiedemann-Franz law

At room temperature the conduction electrons in metal have a much greater mean free path than the phonons. For this reason, heat conduction in metals is primarily due to the electron. The Wiedemann-Franz law was experimentally established in 1853, and according to it, it is verified:

$$\frac{\kappa}{\sigma} = LT \quad (11)$$

Where L is the Lorenz number, which is given as function of some universal constants, since

$$L = \frac{\pi^3}{3}\frac{k_B^2}{e^2} = 2.44\cdot 10\text{-}8 \text{ W } \Omega \text{ K}^{-2} \quad (12)$$

Where k_B is the Boltzmann constant of the gases and e is the electron charge.

3 Bibliography

- **C. Kittel, Introduction to Solid State Physics, 8th edition**
- **N.W: Ashcroft, N.D. Mermin, Solid State Physics, (1976), pp. 6-15.**

In the WEB

https://www.electrical4u.com/thermal-conductivity-of-metals/
http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/thrcn.html

https://courses.lumenlearning.com/physics/chapter/22-6-the-hall-effect

4 Equipment

4.1 Thermal conductivity measurements

1. Calorimeter vessel, 500 ml (heat reservoir)
2. Calorimeter vessel, 500 ml (cold Reservoir) with magnetic stirring bar
3. Copper rod ($L = 0.315$ m; cross-section, $S = 4.91 \cdot 10^{-4}$ m^2)
4. Digital temperature meter
5. Temperature probe, immersion type, Pt10
6. Immers. heater, 300W, 220-250VDC/AC
7. Magnetic stirrer without heating
8. Support rod, stainless steel, 1000 mm (for the copper rod)
9. Support rod, stainless steel, 750 mm (for the temperature probes)

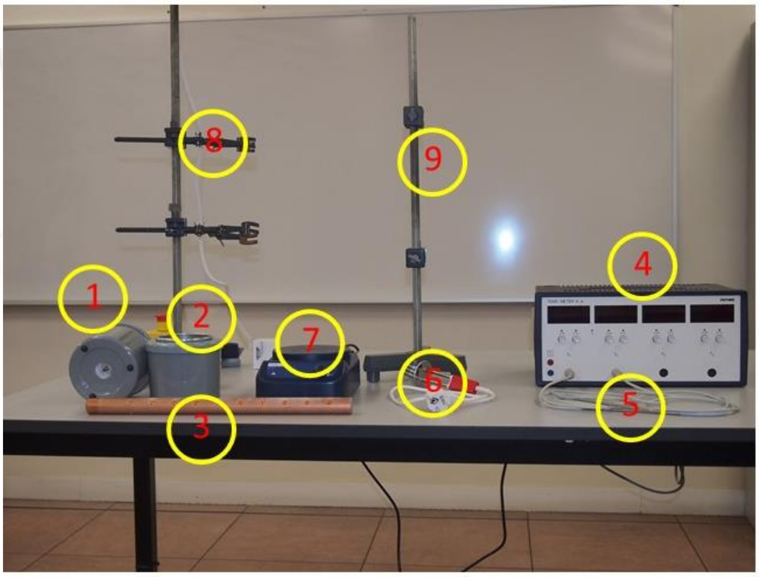


Figure 3. Equippment for the experiment of the thermal conductivity measurements.

4.2 Electrical conductivity measurements

1. Power supply
2. Measuring amplifier
3. Digital multimeter (Amperimeter)
4. Digital multimeter (Voltimeter)
5. Copper rod (L = 0.315 m; cross-section, S = $4.91 \cdot 10^{-4}$ m^2)
6. Cables

Figure 4. Equippment for the experiment of the electrical conductivity measurements.

5 Experimental procedure

5.1 Measurements of the thermal conductivity

5.1.1 Measurement of the heat capacity of the lower calorimeter (cold reservoir).

There different methods for determining the heat capacity of the calorimeter. One of them consists of pouring hot water into the calorimeter, for instance 200 ml ($m = m_2$); the calorimeter is closed and the temperature is measured ($T = T_1$). Some minutes later, a certain amount of cold water, for instance 200 ml ($m = m_2$) is introduced in the calorimeter with a known temperature ($T = T_2$). The system is allowed to evolve until thermal equilibrium has been achieved. Then, the heat capacity of the calorimeter is given by the expression:

$$C = \frac{c_w\left[m_1(T_1 - T) + m_2(T_2 - T)\right]}{(T - T_1)} \quad (13)$$

However, in this practice, the heat capacity given by the manufacturer will be used. The heat capacity of the lower calorimeter is $C = 78 \pm 20$ J K^{-1}.

5.1.2 Determination of the influence of the surroundings.

As it has been indicated in the theoretical background, to determine the thermal conductivity coefficient, it is necessary to find out the heat per unit of time deliver by the bar to the cold reservoir in the stationary regime. However, the environment can also give heat to the cold reservoir during the experiment. In order to evaluate this, it is necessary to follow the following procedure:

a) Weigh the lower calorimeter at room temperature
b) Pour around 300 ml of water into the calorimeter
c) Introduce the stirring magnetic bar inside the calorimeter
d) Introduce ice into calorimeter
e) Put the calorimeter onto the magnetic stirring (7). Switch on the magnetic stirring so that the magnetic bar start turning around.
f) Measure the temperature of the water plus ice by using the experimental set-up of the figure 4 (switch on the temperature meter).
g) Wait up to the temperature in the calorimeter is 0 ºC.
h) Once the temperature is 0 ºC, take out all the ice which is present inside the calorimeter and start to measure the temperature of the calorimeter. Take a value every minute.
i) Take measurements during 20 minutes.
j) On finishing the measurements, weigh the calorimeter with the water (take out the magnetic stirring) and determine the mass of water inside the calorimeter (m_{w1}).
Remark: Before starting to measure the temperature, prepare a table with the values for the time.

Figure 5. The lower calorimeter and the magnetic stirring. Experimental set up for measuring the influence of the surroundings.

5.1.3 Make a plot with temperature ($T-T_0$) (in fact T_0 = 0 ºC) versus time. Perform a least square fitting of the curve
It is verified:

$$Q_{surr} = \left(m_{w1}c_w + C\right)\Delta T \Rightarrow \frac{\partial Q_{surr}}{dt} = \left(m_{w1}c_w + C\right)\frac{dT}{dt} \quad (14)$$

The value of the slope of the curve is $\frac{dT}{dt}$. In this way it is possible to evaluate the influence of the surroundings.

5.1.4 Determination of the heat flow through metal rod thermal conductivity

5.1.5 *Methode*:

1. Perform the experimental set-up according to Fig. 6.
2. Weigh the empty, lower calorimeter.
3. Insert the insulated end of the metal rod into the upper calorimeter vessel. To improve the heat transfer, cover the end of the metal rod with heat-conduction paste
4. Attach the metal rod to the support stand in such a manner that the lower calorimeter can be withdrawn from beneath it.
5. The height of the lower calorimeter can be changed with laboratory jack. When doing so, care must be taken to ensure that the non-insulated end of the rod remains completely immersed in the cold water during the experiment.
6. The temperature probe must be positioned as close to the rod as possible.
7. The outermost indentations on the rod are used to measure the temperature difference in the rod. To improve the heat transfer between the rod and the temperature probe, use heat conduction paste.
8. Pour water in both calorimeters: upper calorimeter (hot reservoir; nearly filled of water, around 500 ml); lower calorimeter (cold reservoir; around 300-400 ml)
9. Put the immersion heater in the upper calorimeter
10. Introduce ice in the lower calorimeter. Switch on the magnet stirring (the magnet rod must be inside the calorimeter). In this way the temperature in the lower calorimeter is homogenized.
11. Start to heat the upper calorimeter. Ensure that the upper calorimeter is well
12. Keep the water in the lower calorimeter at 0°C with the help of ice.
13. Watch the values of the temperature of the copper rod given by the two temperature probes. The measurements can be begun when a constant temperature gradient has been established between the upper and lower surface probes. It means that the values of the temperature given by both temperature probes hardly change (it takes around 15-20 minutes).
14. Write the values of the temperature of the copper road. These values will be used for determining $\frac{dT}{dx} = \frac{T_2 - T_1}{x_2 - x_1}$ (x_2-x_1) is the distance between both points in the rod).
15. Before starting the measurements prepare a table where time versus time will be noted (write the values for the time; you should take one measurement per minute).
16. Take out all the ice of the lower calorimeter (cold reservoir)
17. Put one of the temperature probe in the lower calorimeter
18. Start taking the value of the temperature of the water of the cold reservoir, one value per minute, during 20 minutes. Watch from time to time that the hot reservoir is at 100 ºC (you see water vapour going out from the calorimeter) and the heater is immersed in the water.
19. When you have finished, reweigh the lower calorimeter with the water (without the magnet rod). You must determine the mass of water inside the calorimeter, m_{w2}.

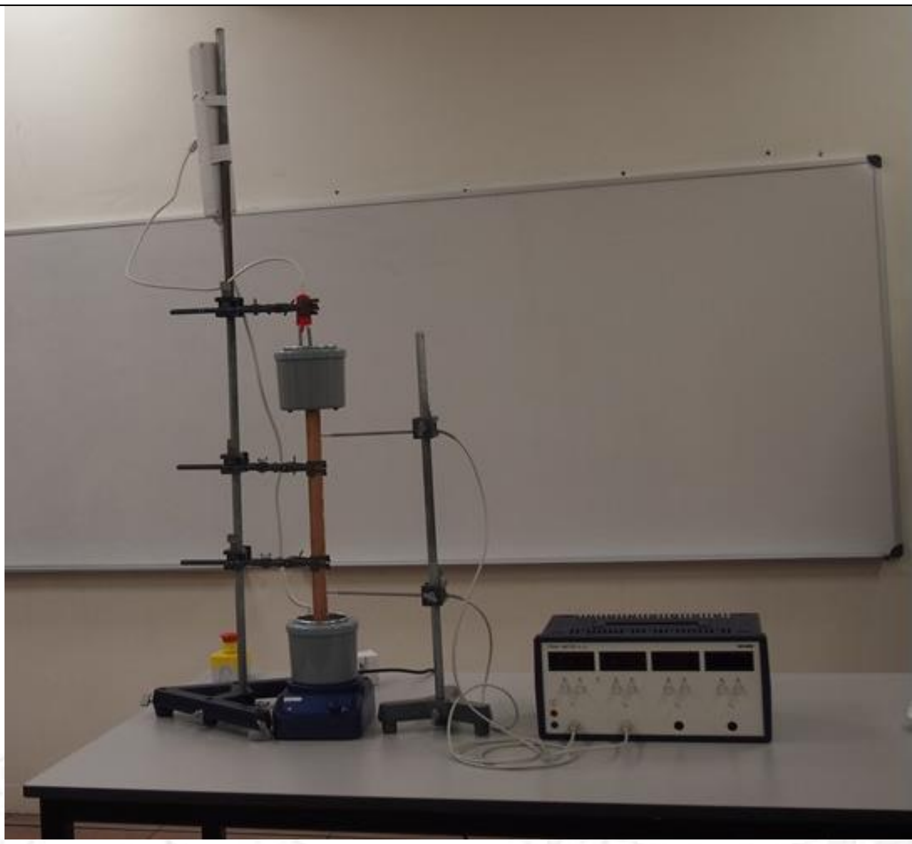


Figure 6. Experimental set-up for the measurement of the thermal conductivity

5.1.6 Make a graphic with temperature (y-axis) versus time (x-axis). Perform a least square fitting of the straight line. Find out the value of the straight light.
The heat per unit of time delivers to the calorimeter (+ surroundings) is determined as:

$$Q_{Net} = \left(m_{w2}c_w + C\right)\Delta T \Rightarrow \frac{\partial Q_{Net}}{dt} = \left(m_{w2}c_w + C\right)\frac{dT}{dt} \quad (15)$$

5.1.7 Now you can determine the coefficient of thermal conductivity, κ, since:

$$\frac{\partial Q}{dt} = \frac{\partial Q_{Net}}{dt} - \frac{\partial Q_{Sorr}}{dt} = JS = -\kappa S\frac{dT}{dx} \quad (16)$$

5.1.8 Compare the experimental value with the corresponding value that you can find in the bibliography (value at room temperature)

5.2 Measurements of the electrical conductivity

For measuring the electrical conductivity of the copper rod it will be used an experimental set-up that is slightly different to the one given in Figure 2. In this case, it is possible to vary the intensity of the circuit from the power supply and it will not be necessary to include a variable resistance in the circuit. The experimental set-up is given in Figure 7. In this experiment the potential difference between two points of the copper rod bar will be measured as a function of the intensity of the electric current flowing through it; The potential difference is very small, so it is necessary to amplify the potential signal.

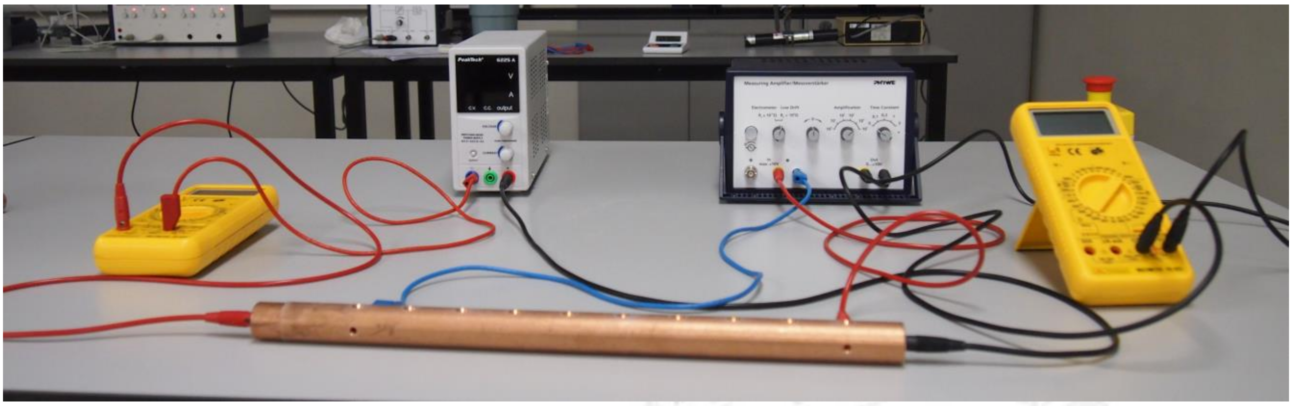


Figure 7. Experimental set-up for measuring the electrical conductivity

With the power supply, amplifier and mutimeters switch off perform the following connections:

5.2.1 Connect control (1) of the power supply to the control (6) of the amperimeter. The control (7) of the amperimeter to the point (8) of the copper rod. From the point (9) of the copper rod to the control (2) of the power supply. In this way, the power supply, the amperimeter and the copper rod are connected in series (Figure 8).

5.2.2 Connect the point (10) of the copper rod to the control (16) of the amplifier Figure 9). Subsequently connect the point (11) of the rod to the control (17) of the amplifier. In this way the amplifier is connected in parallel to the copper rod for measuring the potential difference between two points of the bar. Let us point out that the distance between the points (10) and (11) of the copper rod is d = 0.315 m.

5.2.3 For measuring the voltage, let us connect the voltimeter to the amplifier (the signal is amplified). Connect the control (20) of the voltimeter to the control (18) of the amplifier (ground) and the point (21) of the voltimeter to the point (19) of the amplifier.

5.2.4 The amperimeter must be in the scale 20m/20 A in DC mmode, since direct electric currents between 0-5 A will be measured. The voltimeter must be in he scale 2 in the DC mode.

5.2.5 Switch on the power supply, the amplifier, the amperimeter and the voltimeter.

5.2.6 In the power supply, with the control (3) off (there is no light in the part of the screen marked as output), move the control of the intensity (5) to zero (turn to the left). Then move the control of the voltage (5) to 5 V.

5.2.7 In the amplifier. Check that control (12) is at 10^4 Ω. Put control (14) to 10^4 (amplification); it means that the voltage measured in the voltimeter is multiplied by 1·104. Put the control (15), time constant to zero.

5.2.8 In principle there is no current flowing through the circuit, so the potential difference between the ends of the bar is zero, and the reading in the voltimeter is zero. IF this is not the case, move the control (13) of the amplifier until the reading is zero. The control (13) is very sensitive, so you must be patient. If it is very close to zero, it is ok…

5.2.9 Now you are ready to initiate the experiment. Push the control (3) of the power supply, marked as output, and then you can vary the value of the intensity, control (5) of the power supply. You must see in the screen of the power supply two red lights in CC and output. With the control (5) varies the intensity of the current in steps of 0.4 A from zero up to 4.5 A. For each value note the value of the voltage that appears in the voltimeter. Remember: this value is multiplied by 10^4 (the signal has been amplified).

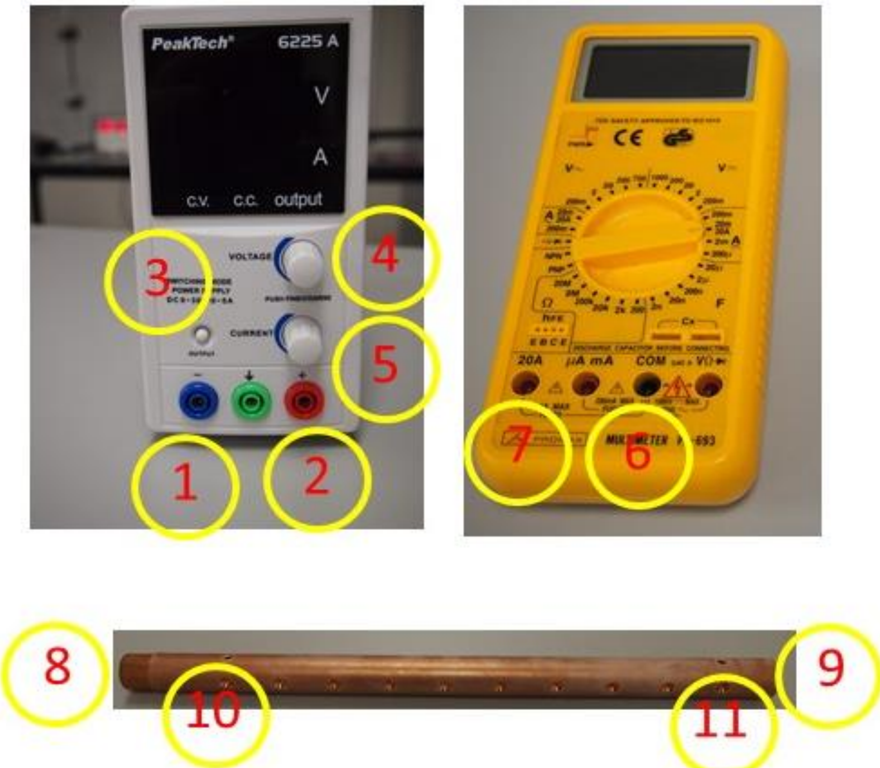


Figure 8. Experimental set-up. Power supply, amperimeter and copper rod

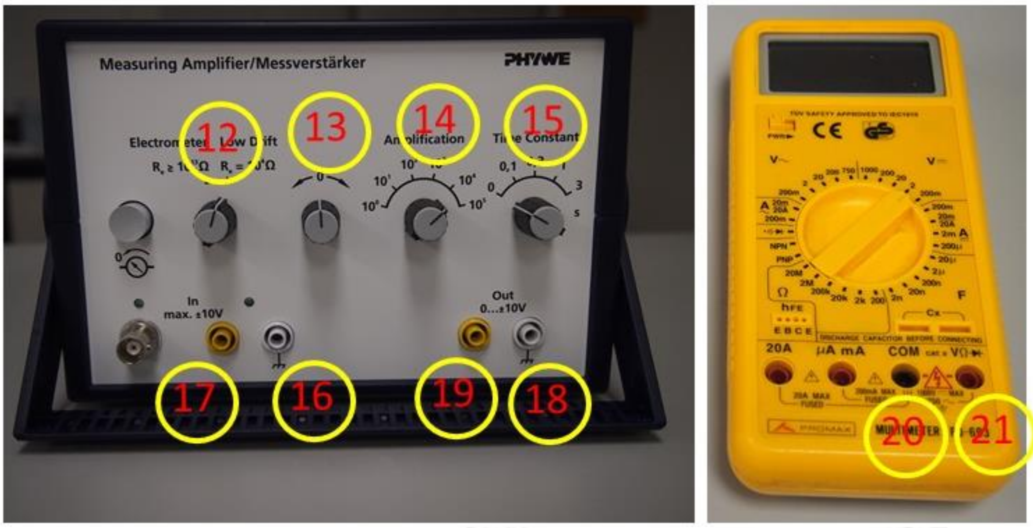


Figure 9. Experimental set-up. Power supply, amperimeter and copper rod

5.2.10 Make a table with the intensity versus voltage. Do a plot with the data (the x-axis is the intensity and the y-axis is the voltage) and perform a least square fitting of the straight line. By using the expression 10, it is possible to obtain the electrical conductivity of the copper rod. Remember: $L = 0.315$ m and $A = 4.91 \cdot 10^{-4}$ m).

5.2.11 From the values of the coefficient of the thermal conductivity and the electrical conductivity determine the experimental value of the Lorenz number ($T = 300$ K). Compare this value with the theoretical one discuss the results.