# Susceptibility measurements

## Theoretical background

The magnetic susceptibility is a physical quantity that is usually used to characterize the magnetic behaviour of a material. It is defined by the expression

$$\chi = \frac{M}{H} \quad (1)$$

Where $M$ is the magnetization and $H$ is the magnetic field.

The magnetic susceptibility measurements are performed on applying a small magnetic field H and measuring the value of the magnetization that is induced in the material. The measurements are carried out in a certain range of temperature. It is very usual to choose a temperature range going from 1.5 K up to room temperature, or higher if the ordering temperature is at high temperatures.

In case the material is not magnetic and it only contains a small amount of magnetic atoms, the distance between the magnetic atoms is too large and there are not magnetic interactions among their magnetic moments. Then, in the absence of a magnetic field, the magnetic moments of the atoms are randomly orientated. On applying a small magnetic field, a certain magnetization in the direction of the magnetic field appears and the thermal evolution of the susceptibility is given by the Curie law:

$$\chi = \frac{C}{T} \quad (2)$$

Where C is the Curie constant. Its value is given by the expression

$$C = \frac{N\mu_0\mu_{ef}^2}{3k_B} \quad (3)$$

Where $N$ is the number of magnetic atoms, $\mu_0$ is the vacuum permeability, $k_B$ is the Boltzmann constant and $\mu_{ef}$ is the effective magnetic moment of the magnetic atoms. The effective magnetic moment is given by the expression:

$$\mu_{ef} = g\mu_B\sqrt{J(J+1)} \quad (4)$$

Where $g$ is the Landé factor, $\mu_B$ is the Bohr magneton and $J$ is the quantum angular moment.

The thermal evolution of susceptibility and the inverse of the susceptibility curves are given in Figure 1. This thermal evolution is characteristic of a paramagnetic material.

In magnetic materials the distance between the magnetic atoms is of the order of the atomic lattice, and in this case the magnetic interaction between the magnetic atoms define the magnetic behaviour of the material. Besides the magnetic interactions that try to order the magnetic moments of the atoms, there is the thermal agitation that tends to orientate the magnetic moments in random directions. In case the magnetic interactions are ferromagnetic, below the ordering temperature, $T_C$ (Curie temperature), the moments tend to be orientated in the same direction. On applying a magnetic field, at $T_C$, it is observed an important increase of the magnetization. Above the ordering temperature, $T_C$, the thermal agitation interactions predominate and there is not a long-range magnetic ordering. In this case, the susceptibility is inversely proportional to the temperature, as it is observed in Figure 2. If the inverse of the susceptibility is represented versus the temperature, a linear behaviour is observed (see Figure 2). The thermal evolution of the inverse of the susceptibility follows the Curie-Weis law given by

$$\chi = \frac{C}{T-\theta} \quad (5)$$

C is the Curie constant and its value is given by the expression (3). $\theta$ is the paramagnetic Curie temperature and its value depends on the nature of the magnetic interactions. If $\theta$ is positive, it means that the magnetic interactions are ferromagnetic.

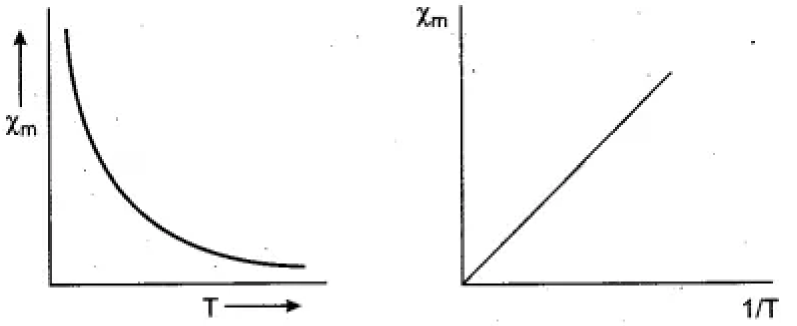

Figure 1. Thermal evolution of the susceptibility and the inverse of the susceptibility for a paramagnetic material.

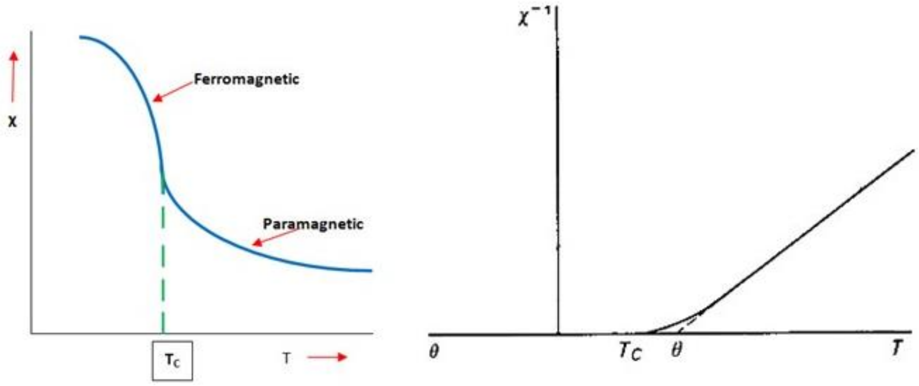

Figure 2. Thermal evolution of the susceptibility and the inverse of the susceptibility for a ferromagnetic material.

For an antiferromagnetic material, the magnetic interactions between the neighbour atoms tend to orientate their moments in opposite directions. The thermal evolution of the magnetic susceptibility is represented in Figure 3 for a polycrystalline material. At the temperature order, $T_N$ (Neel temperature), a maximum in the susceptibility is observed. Above the ordering temperature, the inverse of the susceptibility exhibits a linear behaviour with respect to the temperature (see Figure 3) and it follows the Curie-Weiss law given by the expression (5). However, in this case the paramagnetic Curie temperature, $\theta$ is negative, indicating that the magnetic interactions between the atoms are antiferromagnetic.

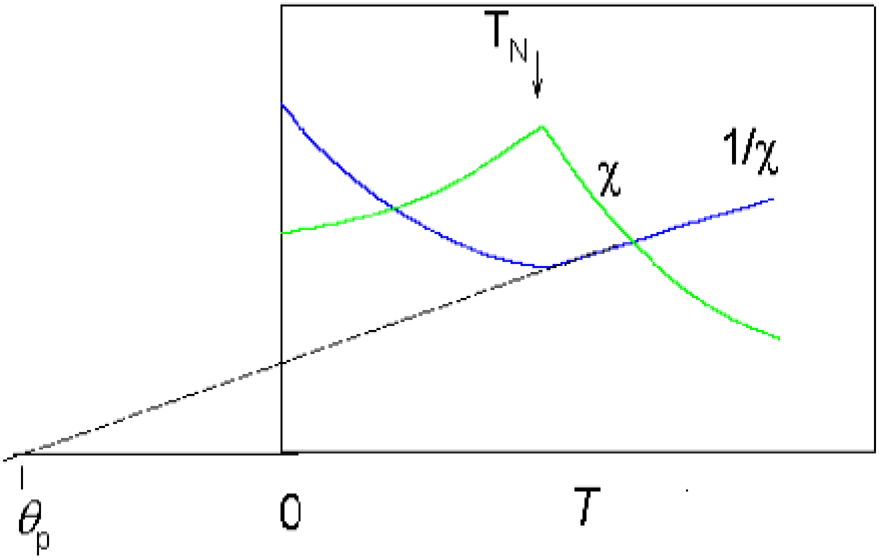

Figure 3. Thermal evolution of the susceptibility and the inverse of the susceptibility for an antiferromagnetic material.

**Comments about magnetic units**

In the International system of units, it is verified:

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \quad (6)$$

It means that H and M have the same units ($Am^{-1}$)and the susceptibility has no units in the internal system of units. Traditionally, in magnetism, it is very usual to work in the Gauss-cgs system. In this case equation (6) is written as:

$$\vec{B} = \vec{H} + 4\pi\vec{M} \quad (7)$$

So, in this units system $\mu_0 = 1$; H is expressed in Oe and M in Oe or emu·$cm^{-3}$. Then the units of the susceptibility are usually expressed as emu·$mol^{-1}$·$Oe^{-1}$ (molar susceptibility). When this is the case, the better option is to work in the Gauss-cgs system of units. Then the Curie constant is written as:

$$C = \frac{N_A \mu_{ef}^2}{3k_B} \quad (8)$$

The total number of magnetic moments is substituted by the Avogadro´s number. Usually the important data are the effective magnetic moment, $P_{ef}$, which is expressed in function of $\mu_B$, and $\theta$, that is expressed in Kelvin. So:

$$P_{ef} = \frac{\mu_{ef}}{\mu_B} \quad (9)$$

In the Gauss-cgs system of units: $k_B = 1.3807 \cdot 10^{-16}$ erg·K$^{-1}$ and $\mu_B = 9.2741 \cdot 10^{-21}$ emu·G$^{-1}$.

**Goal of the practice**

The goals of this practice are to obtain the following information from the susceptibility curve of $LuVO_3$:

-The ordering temperature

-The type of magnetic ordering: ferromagnetic antiferromagnetic

-Fit the inverse of the susceptibility versus the temperature by a straight line and determine from the fitting the effective magnetic moment and the Curie temperature

Compare the experimental effective magnetic moment with the theoretical one (Help: For this part, it is necessary to read the notes of lesson 7, “Magnetic Materials” that you can find in Aula Global, in particular subsection 7.2.2)

**Data for the practice**

*Magnetic susceptibility of $LuVO_3$ versus temperature*

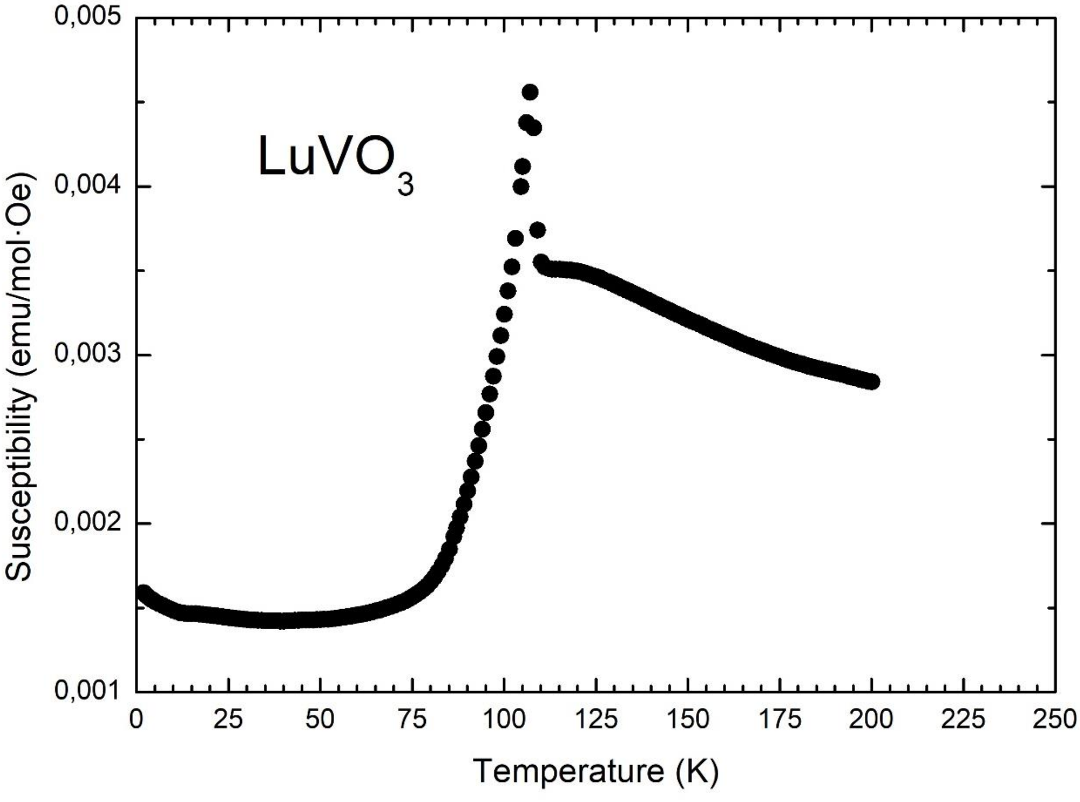

*Table with data: inverse of the susceptibility versus temperature*

| T (K) | $1/\chi$(mol·Oe/emu) |
|---|---|
| 240,17 | 379,76 |
| 245,22 | 383,4 |
| 250,17 | 387,04 |
| 255,15 | 390,55 |
| 260,16 | 394,39 |
| 265,15 | 398,14 |
| 270,16 | 401,68 |
| 275,15 | 405,31 |
| 280,19 | 408,64 |
| 285,21 | 412,52 |
| 300 | 424,35 |

**Work to do**

1) From the Figure corresponding to the susceptibility versus temperature determine the ordering temperature of $LuVO_3$
2) The magnetic order is ferromagnetic or antiferromagnetic?
3) Fit the curve Inverse of the susceptibility ($1/\chi$) versus the temperature with the data given in the table.
4) From the fitting performed in the point 3), what is the value of the Curie temperature, $\Theta$? Is it coherent with the answer given in point 2), What is the value of the Curie constant? From the value of the Curie constant determine the value of the effective magnetic moment, in Bohr magnetons ($\mu_B$).
5) In $LuVO_3$ the only magnetic atom is V. The electronic configuration of the $V^{3+}$ ion is $3d^2$. Determine the theoretical effective magnetic moment from equation (4). The g(LSJ) factor is:

   $$g = \frac{3}{2} + \frac{1}{2}\left[\frac{S(S+1) - L(L+1)}{J(J+1)}\right]$$

   Carry out the calculation: first on coupling the orbital, L, and spin, S, angular momentums ( J = L+S or J = L-S); second considering L = 0, so J = S. (Note: in the transition elements (such as V), because of the crystal field the angular momentum L is quenched and the magnetic moment is only defined by S).

6) Is there a good agreement between the experimental effective magnetic moment (obtained in the fitting) and the theoretical one?

**Straight line fitting:**

$$slope=\frac{n\sum_i x_i y_i-\left(\sum_i x_i\right)\left(\sum_i y_i\right)}{n\sum_i x_i^2-\left(\sum_i x_i\right)^2};\ \operatorname{int}ercept=\frac{\left(\sum_i x_i^2\right)\left(\sum_i y_i\right)-\left(\sum_i x_i\right)\left(\sum_i x_i y_i\right)}{n\sum_i x_i^2-\left(\sum_i x_i\right)^2}$$

$$slope\ \text{error}=S*\sqrt{\frac{n}{n\sum_i x_i^2-\left(\sum_i x_i\right)^2}};\ S=\sqrt{\frac{\sum_i (y_i-ax_i-b)^2}{n-2}}$$