BALMER SERIES: DETERMINATION OF RYDBERG'S CONSTANT

1 AIM

- To determine the grating constant, g, of a diffraction grating by observing the known spectral lines of the Balmer series of Hg.
- To determine the wavelengths, λ, and the Rydberg's constant, R_y, after examining the spectral lines of the Balmer series of H.

2 THEORETICAL BACKGROUND

Inside a heated spectral tube, due to the ionization processes caused by collisions, molecular H_2 is converted to atomic hydrogen. Electrons from the H atoms are excited to higher energy levels, due to the collisions with free electrons with enough energy. **When the excited electrons return to the lower energy levels, light is emitted with a frequency** ν, which is given by the difference of energy between the two concerned states according to:

$$h\nu_{nm} = \Delta E = E_m - E_n \tag{1}$$

Being $h = 6.62 \times 10^{-34}\ m^2\ kg/s$ the Planck's constant. Quantum mechanics predicts that the energy of the permitted levels of a H atom is given by:

$$E_n = -\frac{1}{8}\,\frac{e^4 m_e}{\epsilon_0^2 h^2}\,\frac{1}{n^2},\ n = 1, 2, 3, ... \tag{2}$$

Here, n is the principal quantum number of the energy level, $\epsilon_0 = 8.8542\times10^{-34}\ As/Vm$ is the vacuum dielectric constant, $e = 1.602 \times 10^{-19}\ C$ is the electron charge and $m_e = 9.109 \times 10^{-31}\ kg$ is the electron mass at rest. Therefore, when electrons return to lower energy levels the emitted light can only have the following frequencies, which characterize the H spectrum:

$$\nu_{mn} = \frac{1}{8}\,\frac{e^4 m_e}{\epsilon_0^2 h^3}\left(\frac{1}{n^2} - \frac{1}{m^2}\right),\ n, m = 1, 2, 3, ... \tag{3}$$

Where n is the principal quantum number of the lower energy level, and m is the principal quantum number of the upper energy level. This expression is often expressed in terms of the inverse wavelength, $\lambda^{-1} = \nu/c$, being $c = 3 \times 10^8\ m/s$ the light speed:

$$\frac{1}{\lambda_{mn}} = R_y\left(\frac{1}{n^2} - \frac{1}{m^2}\right),\ n, m = 1, 2, 3, ... \tag{4}$$

Where R_y is the Rydberg's constant, that is equal to:

$$R_y = \frac{1}{8}\,\frac{e^4 m_e}{\epsilon_0^2 h^3 c} = 1.097 \times 10^7 \; m^{-1} \tag{5}$$

Different spectral line series exist (see Figure 1) depending on the value of n, the principal quantum number of the lower energy level:

- $n = 1$, Lyman series. Spectral range: Ultraviolet.
- $n = 2$, Balmer series. Spectral range: Ultraviolet and visible.
- $n = 3$, Paschen series. Spectral range: Infrared.
- $n = 4$, Bracket series. Spectral range: Infrared.
- $n = 5$, Pfund series. Spectral range: Infrared.

It turns out that only the first few lines of the Balmer series fall within the visible spectral range, which extends from about 380 to 750 nanometers.

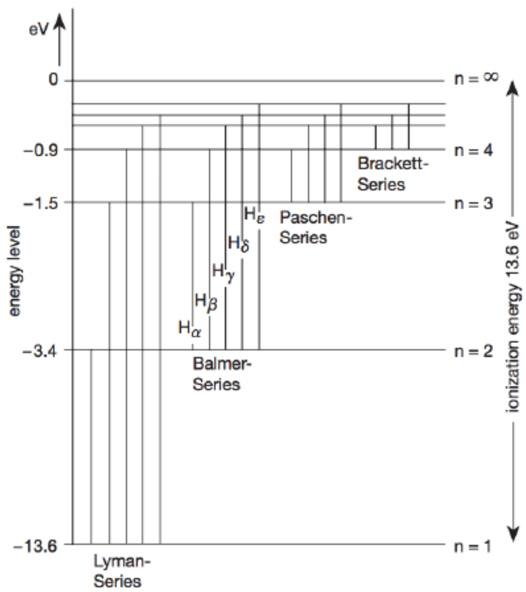

Figure 1: Hydrogen spectral series.

Atomic emission spectra similar to this one of the atomic hydrogen also appear for atoms similar to hydrogen. As the energy levels are characteristics of each atom, **each element's emission spectrum is unique**.

3 DESCRIPTION OF THE EXPERIMENT

In this lab, we will use a diffraction grating to resolve the different visible spectral lines associated with the atomic emission spectrum of both Hg and H gasses. A diffraction grating is an optical element made of a large number of parallel and closely spaced slits, characterized by a grating constant g (i.e., the number of lines/slits per unit length). When light of a certain wavelength λ impinges on a grating, it is diffracted. Intensity peaks beyond the grating will occur when the angle of diffraction α fulfills the Bragg condition:

$$k\lambda = g \; sin\alpha, \;\; k = 0, 1, 2, 3, ... \tag{6}$$

Where the integer k gives the order of the diffraction, with $k = 0$ corresponding to no diffraction. Because of the large number of slits in a diffraction grating, the intensity peaks observed through it are sharp and narrow, providing a high resolution for spectroscopic applications.

The ionized spectral tube has to be aligned with the diffraction grating and separated by a distance d. When the observer places their head in front of the grating, light is collected by their eye on the retina. The spectral lines of wavelength λ that cause the diffraction will be observed on the position of the meter scale aligned with the prolongation of the light beam, at both sides of the spectral tube and separated between them by a distance $2\ l$ (see Figure 2). For the diffraction of the k-th order, the following relation derived from the Bragg condition is deduced from the geometry of the setup:

$$k\lambda = \frac{gl}{\sqrt{d^2 + l^2}}, \ \ k = 0, 1, 2, 3, ... \tag{7}$$

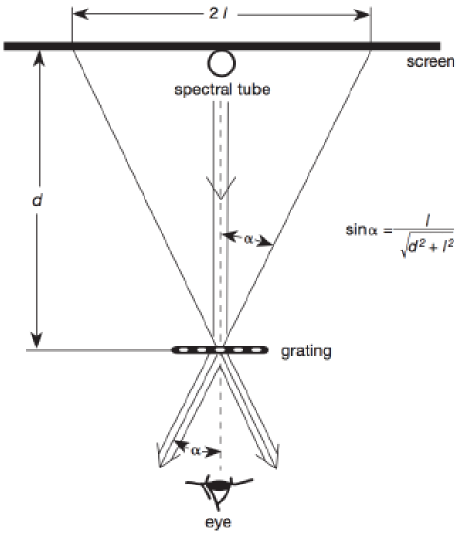

Figure 2: Diffraction of light by a grating.

4 EXPERIMENTAL SET UP AND PROCEDURE

The experiment for the observation of the atomic spectra is formed by:

- Spectrum tube, Hg.
- Spectrum tube, H.
- Holders for the spectral tubes.
- Covers for the spectral tubes.
- Diffraction grating.
- High voltage supply unit.
- Meter scale, 1 m.
- Connection cables.
- Ruler.

Proceed now using the following steps:

1. Hg spectral tube will be already mounted in the holder and connected to the high-voltage power supply. The meter scale is attached directly behind the spectral tube to minimize parallax errors. The luminous capillary tube should be visible through the grating; check that the slit in the cover tube is facing out.

2. The diffraction grating should be set up at about 30 cm away from the tube and parallel to it; measure the distance between the grating and the scale, d. Check that the grating is located at the same height as the spectral tube.

3. Now, the Hg spectral tube will be heated by adjusting the power source until the gas in the capillary tube starts to glow steadily. Reduce the voltage to the minimum required.

4. Place your head in front of the diffraction grating, at the same height, and look through it without turning your head. Move the scale sliders to mark the position of the spectral lines of the same color that appear on the right and left, and note down the corresponding positions. The room should be sufficiently dark so that spectral lines can be seen while still being able to read the scale. Three lines should be visible for the Hg tube, $m = 1$, $m = 2$, and $m = 3$.

5. Repeat the experiment twice to reduce the random error (each time, a different member of the pair must perform the observation).

IMPORTANT: ASK YOUR TEACHER OR LAB TECHNICIAN TO REPLACE THE SPECTRAL TUBE FOR YOU!!

6. With the H tube mounted in the setup, repeat all the previous steps. In this case, you should easily identify two lines ($m = 1$ and $m = 2$), and hopefully also a third one ($m = 3$), although the latter might require a sufficiently dark environment.

5 TASKS AND QUESTIONS

- Collect all data needed according to the procedure described above.
- Use the first set of measurements (the one corresponding to the Hg tube) to determine the grating constant, g. Take into account that the corresponding wavelengths of the observed Hg lines are:

Color of the spectral line	Wavelength ($\lambda[nm]$)
yellow	578.0
green	546.1
blue	434.8

- Use the second set of measurements (the one corresponding to the H tube) and the value of g you have calculated to determine the experimental value of the wavelengths for the first three Balmer lines observed. Compare it with the theoretical values that are obtained through equation (4).
- Calculate the value of the Rydberg's constant using the experimentally determined wavelengths and compare it with the theoretical value.
- Discuss the accuracy of the obtained values and the possible sources of error.