LABORATORY OF MECHANICS AND RELATIVITY

SEMINAR ON RELATIVITY:

RELATIVISTIC COLLISIONS

1 Aim

- Study the laws for the collisions between relativistic particles
- Show some of the experimental techniques used to analyze this kind of collisions
- Demonstrate, using these techniques, that the relativistic definitions of linear momentum and energy are correct
- Illustrate the differences between the classical and relativistic descriptions of collisional processes

2 Material

- Photographs
- Protractor and graduated rule
- Greaseproof paper

3 Introduction

In this seminar, we will analyze the laws describing the collisional processes between particles with velocities close to the speed of light. To study these processes we must use the Special Theory of Relativity, and they are called *relativistic collisions* to distinguish them from the collisions at low velocity in comparison with the speed of light, which are commonly called *classical collisions*.

In a collision, two bodies approach each other, collide and separate. Before the collision, at a very large distance, the bodies move with constant velocity. After the collision they also move with constant, but different velocities. Usually, we want to know how will be the final state of the bodies (velocity, linear momentum, energy,...) when their initial state and the characteristics of the collision are known. As we are going to show in this seminar, we usually use the *linear momentum and energy conservation laws*:

(a) *conservation of the linear momentum*, according to which the total linear momentum before and after the collision must be the same:

$$\vec{P}_i = \vec{P}_f, \tag{1}$$

where $\vec{P}_i$ is the total linear momentum before the collision, given by the sum of the linear momentum of each of the particles before the collision; $\vec{P}_f$ is the total linear momentum after the collision,

(b) *conservation of the total energy:* the total energy before the collision (kinetic energy + internal energy of the particles) must be equal to the total energy after the collision:

$$E_i = E_f, \tag{2}$$

where E_i is the total energy of the system before the collision, sum of the energies of each of the particles before the collision; E_f is the total energy after the collision.

We can consider two main cases:

- *Ellastic collisions:* the internal energy of the colliding particles does not change as a result of the collision. Hence, as the total energy is given by the sum of the internal energy plus the kinetic energy and must be conserved, this means that *in an elastic collision the kinetic energy is conserved*:

$$T_i = T_f, \tag{3}$$

  where T_i is the total kinetic energy before the collision, sum of the kinetic energies of each of the particles before the collision, and T_f is the total kinetic energy after the collision.

- *Inelastic collisions:* in this case, a fraction of the initial kinetic energy of the system is used to change the internal energy. This happens for example when, as a consequence of the collision, one of the bodies is deformed: part of the initial kinetic energy is used for the deformation of one of the bodies, increasing its internal elastic energy. The total kinetic energy is conserved (kinetic energy + internal energy) but the kinetic energy decreases. The amount of lost kinetic energy depends on the degree of inelasticity of the collision. An special case of inelastic collision occurs when the bodies remain stuck after the collision: in such a case we say that we have a *fully inelastic collision*.

When the velocities of the colliding particles is very small in comparison with the speed of light (*classical collision*), the collision process is described by these conservation laws using the classical definitions of linear momentum, $\vec{p}$, and kinetic energy, T:

$$\vec{p} = m\vec{v}, \tag{4}$$

$$T = \frac{1}{2}mv^2 = \frac{p^2}{2m}, \tag{5}$$

where $\vec{v}$ is the particle velocity.

However, when the particle velocities approach to the speed of light (*relativistic collisions*), the classical description of the collisions, using the classical definitions of linear momentum and energy, are not valid anymore. The conservation laws of energy and linear momentum are still valid but the definitions of linear momentum, kinetic energy, etc, must be modified. The Special Theory of Relativity provides the correct definitions of these dynamical magnitudes for the whole range of velocities (from zero, in which they coincide with their classical definition, until velocities close to the speed of light):

- the *linear momentum*, $\vec{p}$, of a relativistic particle of mass m and velocity $\vec{v}$ is defined:

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{6}$$

- *the total energy*, E, of a relativistic particle is given by the sum of the kinetic energy, T, plus an energy mc^2 (c is the speed of light) associated with the mass of the particle and named the particle *rest energy*:

$$E = T + mc^2 \tag{7}$$

  Thus, the mass appears as a form of energy: the total mass before and after the collision can change if the total energy (kinetic energy + rest energy) is conserved, i.e., if it is converted in an equivalent amount of kinetic energy. This is quite different from Classical Mechanics, in which it is assumed that the total mass before and after the collision is the same.

  The total energy of a particle can be written as a function of its linear momentum, $\vec{p}$:

$$E = T + mc^2 = \sqrt{p^2c^2 + m^2c^4} \tag{8}$$

  Hence, the relation between the kinetic energy of a particle and its linear momentum will be given by:

$$T = \sqrt{p^2c^2 + m^2c^4} - mc^2, \tag{9}$$

  while in Classical Mechanics $T = p^2/2m$.

The best evidences for the validity of the conservation laws of energy and linear momentum in relativistic collisions, and hence for the relativistic definitions of linear momentum and energy, come from the field of *High Energy Physics*: elementary particles are accelerated by intense electric fields to high energies and velocities close to the speed of light. The collisions between these particles can be analyzed and, therefore, checked the validity of the conservation laws to describe these processes.

In the following, we will have as main goals:

(a) To know some of the techniques used to detect the trajectories of elementary particles

(b) To learn one of the methods typically used to determine, from the trajectories of the particles, their linear momentum and, hence, their kinetic energy and total energy

(c) To study, using these techniques and applying the conservation laws of energy and linear momentum, processes of collision between relativistic particles

4 Trajectories of Elementary Particles: The Cloud Chamber

The particles of the microscopic physics are too small and move too fast to be detected by our senses. All we know about these particles comes from instruments specifically designed with this aim.

To observe a particle it is required that yields a sizeable effect on a detector. It must interact somehow with the detector. Most of the detectors make use of a process of ionization, in which the detected particle creates a pair of ions when removing an electron from some of the molecules of the detector. Hence, charged particles are the easiest to detect as they act on the electrons of the molecules which they find along their trajectory. In contrast, neutrons are difficult to detect as they do not interact with the electrons.

There are different types of detectors. The detector used will depend on the information that is required to obtain. *The Cloud Chamber* constitutes one of the detectors used to visualize the trajectory of elementary particles. It consists of a recipient filled with air or another gas at a slightly higher temperature than condensation. The chamber is designed in such a way that its volume can be rapidly increased. Such an expansion makes that the gas reaches a temperature below condensation, still being a gas. Part of the gas can now condense, that is, the gas is in a metastable state. The gas condenses more easily if there are charged particles inside. Thus, if the gas is traversed by a charged particle, which yields ionization of some of the gas molecules along its path, the gas will condense on those ions, and the particle will be made visible by means of a trace of droplets of liquid.

In this way, the trajectories of charged elementary particles can be visualized in a cloud chamber and, hence, a cloud chamber can be used to follow the trajectories of the particles during a collision. Taking a picture of these trajectories, we can later on to study in detail the features of such a collision. In Fig. 1, it is shown a picture in a cloud chamber of an ellastic collision between two protons: it can be seen the trajectory of the incident proton (1), and the trajectories of the two protons (2) and (3) after the collision. One of the protons was initially at rest.

It should be remarked that in a cloud chamber is only possible to detect trajectories of charged particles. A neutral particle, like a neutron, leaves no traces in a cloud chamber.

5 Magnetic Analysis of Relativistic Particles

The cloud chamber allows to record the trajectories of the particles but, in principle, it provides no information about their momentum or energy. One of the main methods used with that aim is to analyze the trajectories of the particles in a known magnetic field, which allows to know the sign of the particle as well as its momentum.

In a magnetic field, the trajectory of a charged particle is deflected. The curvature of the trajectories of the particles in the picture shown in Fig. 1 is due to a magnetic field perpendicular to the plane of the sheet.

Such a deviation is due to the fact that a charge q moving in a magnetic field $\vec{B}$ experiences a perpendicular force proportional to its velocity according to

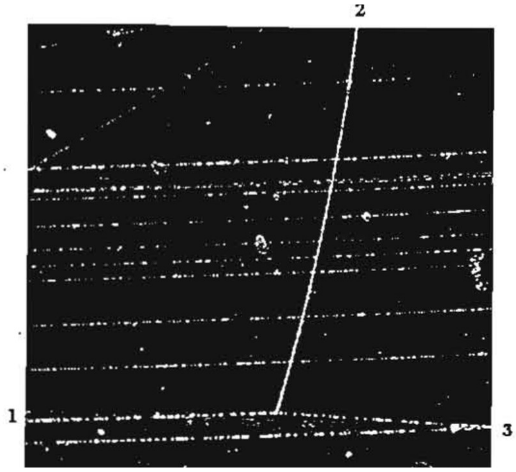

Figure 1:

$$\vec{F} = q\,(\vec{v} \times \vec{B}). \tag{10}$$

Thus, the force on a charged particle due to a magnetic field is always perpendicular to its velocity. A force perpendicular to the velocity yields a change in the direction of the motion, but not a change in the magnitude of the velocity (and so in the magnitude of the momentum), which would require a force along the direction of the motion of the particle. If the particle moves in a uniform magnetic field (equal everywhere) perpendicular to the velocity, the result is that the particle describes a circular trajectory in a plane perpendicular to $\vec{B}$. It can be shown that the relation between the radius of curvature r of this trajectory, the momentum p of the particle and the magnetic field B is given by:

$$p = qBr \tag{11}$$

where q is the electric charge of the particle. It is interesting to notice that this result is the same than in classical Physics except for that in the expression (11) $\vec{p} = \frac{m\vec{v}}{\sqrt{1-v^2/c^2}}$, instead of $\vec{p} = m\vec{v}$.

In many cases, we can be sure that $q = \pm e$ (e is the electron charge), so that if B is known and the radius of curvature r of the trajectory is measured, it is simple to determine the particle momentum. This technique allows in this way to obtain important information about the dynamical state of the particles before and after the collision.

5.1 Determination of the radius of curvature of the trajectory

As it has been explained above, to know the energy of a charged particle in a cloud chamber, it is enough to determine the radius of curvature r of its trajectory in a magneitc field B ($p = qBr$).

Fig. 2 illustrates a simple method to obtain the radius of curvature:

1. Choose two points of the trajectory. The distance of each point to the center of the

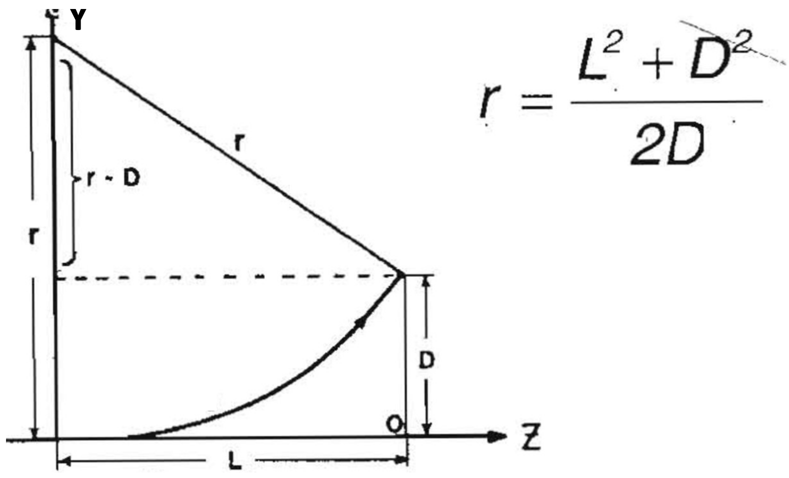

Figure 2:

trajectory must be equal to the radius of curvature r that we want to determine, as indicated in the figure.

2. Next we draw the tangent to the trajectory at the first chosen point (z axis in the figure) and from the second point we draw the perpendicular to the tangent (the intersection point is indicated by O in the figure).

3. Measure the distance L from the first point of the trajectory to the intersection point O and the distance D from the second point of the trajectory to the point O. Applying the Pythagoras theorem to the right triangle with hypothenuse r, and the other two sides L and $r - D$, as shown in the figure, it yields:

$$L^2 + (r - D)^2 = r^2,$$

so that the radius of curvature will be,

$$r = \frac{L^2 + D^2}{2D} \tag{12}$$

Question

As an example of application, we will consider the trajectory shown in Fig. 3, which corresponds to an electron moving in a magnetic field $B = 1.2 \pm 0.02\,\mathrm{T}$ perpendicular to the plane of the figure. The size of the figure is 70% of the actual size of the trajectory.

The electron in the magnetic field must describe a circular trajectory. The spiral shape of the trajectory shown in Fig. 3 is due to the loss of energy of the electron, associated, for example, to the collisions with other particles of the medium, etc, which determines a loss of momentum and, as a consequence, a gradual reduction of the radius of curvature of the trajectory ($r = p/qB$).

- Determine, using the methodology described above, the radius of curvature r corresponding to the initial part of the electron trajectory (the lower part in Fig. 3; do not forget the scale of the figure).

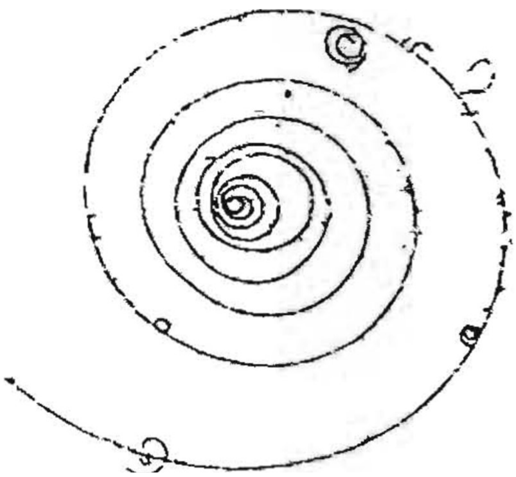

Figure 3:

- Obtain the initial energy and linear momentum of the electron.
- Calculate the velocity of the electron using the classical definition of linear momentum. Why the obtained result is not correct ? Determine the real velocity of the electron using the relativistic definition of linear momentum.

Note: **(i)** express the linear momentum of the electron in units of MeV/c or GeV/c, and its energy in units of MeV or GeV. Remind that $1\,\mathrm{MeV} = 10^6\,eV$, $1\,\mathrm{GeV} = 10^9\,\mathrm{eV}$, and $1\,\mathrm{eV} = 1.6 \times 10^{-19}$ joules; **(ii)** all the measured and calculated magnitudes must be given together with their corresponding uncertainties.

Data: electron charge, $e \equiv 1.6 \times 10^{-19}$ coulombs; $c = 2.9979 \times 10^8\,m/s$; electron mass, $m_e = 0.511\,\mathrm{MeV}/c^2$.

6 Analysis of Relativistic Collisions

Along this Section, we will use the techniques we have described above to analyze collisions betwen particles with relativistic velocities (close to the speed of light).

(A) Proton-proton ellastic collision

We will analyze an ellastic collision between two relativistic protons. We will check the laws governing the collisions between protons and we will demonstrate that a classical analysis of this process leads to wrong results.

The supplied photo shows an ellastic proton-proton collision (Fig. 1) in a hydrogen cloud chamber. The incident proton 1 has a kinetic energy 3 ± 0.06 GeV and collides with a proton at rest. The trajectories of the two protons after the collision are indicated by the numbers 2 and 3. The two trajectories are highlighted in the photo. The trajectories of the protons are curved due to a magnetic field perpendicular to the plane of the figure, $B = 1.70 \pm 0.07$ Teslas, and the radii of curvature of the trajectories 2 and 3 are, respectively: $r_2 = 66.9 \pm 0.5\,\text{cm}$, $r_3 = 7.34 \pm 0.10\,\text{m}$.

- Determine the linear momentum of the incident proton, and use the relation (11) to obtain the linear momenta of the protons after the collision from the curvature of their trajectories in the field B.

- Measure tha angle between the trajectories of the protons after the collision, and check that the total linear momentum before and after the collision is the same. To measure adequately this angle, draw the proton trajectories on the supplied greaseproof paper and measure on it this angle. Notice that the trajectories are curved: to determine correctly the angle, it is convenient to draw tangents to the trajectories at the collision point and measure the angle between them.

- Demonstrate, using the expression (9) for the relativistic kinetic energy, that the kinetic energy is conserved during the ellastic collision between the protons. Check that, using the classical expression for the kinetic energy $T = p^2/2m$, we would wrongly find that the kinetic energy is not conserved in an ellastic collision.

- A second argument that clearly demonstrates the invalidity of the classical description of the collision between the protons is based on the angle between the protons after the collision. As you will have measured before, this angle is lower than 90°. Demonstrate that a classical analysis of an ellastic collision between two identical particles (as the two protons), using the conservation laws of linear momentum and kinetic energy, defined as $T = p^2/2m$, would lead to the conclusion that the angle between the two particles after the collision should be equal to 90°, in disagreement with the measured angle in the photograph.

  Note: express the linear momenta of the particles in MeV/c or GeV/c, and the energy of the particles in MeV or GeV. Use that $1\,\text{MeV} = 10^6\, eV$, $1\,\text{GeV} = 10^9\,\text{eV}$, and $1\,\text{eV} = 1.6 \times 10^{-19}$ joules;

  Data: electron charge, $e \equiv 1.6 \times 10^{-19}$ coulombs; $c = 2.9979 \times 10^8\, m/s$; proton mass, $m_p = 938.3\,\text{MeV}/c^2$.

(B) Pion-proton inelastic collision

In this example, the conservation laws of linear momentum and total energy[1] (kinetic energy + rest energy) will be used to analyze inelastic collisions between relativistic particles.

This process is shown in the photograph that has been supplied with this aim and is illustrated Fig. 4.

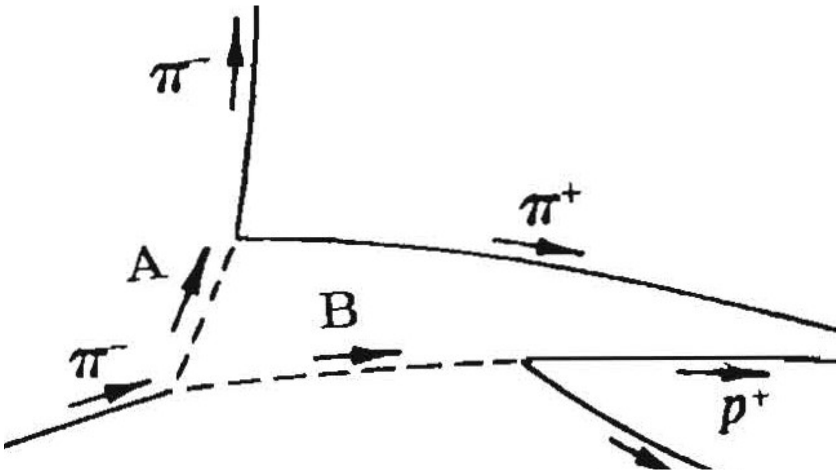

Figure 4:

The photograph shows the collision between a π^- meson and a proton in a cloud chamber of 500 litres of hydrogen. The π^- meson enters in the camera at high speed through the left and collides with a proton at rest. The trajectory of the π^- particle ends abruptly at the point of collision originating two neutral unknown particles, A and B, that being uncharged do not not leave a track in the cloud chamber. These neutral particles, after travelling a certain distance, are disintegrated in pairs of charged particles ($A \to \pi^+ + \pi^-$; $B \to \pi^- + p$), which trajectories are visible in the cloud chamber. Due to a magnetic field perpendicular to the plane of the figure, $B = 1.5 \pm 0.03$ Teslas, the trajectories of these particles are curved and the corresponding radii of curvature are:

$$\begin{array}{lll} A: & r_{\pi^+} = 49.1 \pm 0.5\,\mathrm{cm}; & r_{\pi^-} = 88.7 \pm 0.6\,\mathrm{cm}; \\ B: & r_p = 2.32 \pm 0.04\,\mathrm{m}; & r_{\pi^-} = 11.5 \pm 0.4\,\mathrm{cm}; \end{array}$$

- Determine the linear momenta of the neutral particles A and B, products of the collision (π^-, p), as well as of the particles resulting from the disintegrations of A and B.

[1]Notice that, while in an ellastic collision the kinetic energy is conserved because the rest energy before and after the collision are the same, in an inellastic collision the rest energy changes and so the kinetic energy before and after the collision are not the same. However, the total energy (kinetic energy + rest energy) is always conserved in any type of collision.

- Identify the unknown particles A y B using the table provided at the end of this guide. Notice that masses are given in MeV$/c^2$.

- Determine the kinetic energy of the initial π^- colliding with the proton at rest.

Note: in order to measure adequately the angle between the trajectories of the particles resulting from the disintegrations of A and B, draw these trajectories on greaseproof paper and measure the angles on this paper. Notice that the trajectories are curved: to determine correctly the angle, it is recommended to draw the tangents to the trajectories at the point of the disintegration and determine the angle between them.

(C) Kaon-Proton inellastic collision

In this case, as in the example (B), we will use the conservation laws of the linear momentum and the total energy (kinetic energy + rest energy) to study the inellastic relativistic collision kaon-proton.

The photograph that shows this process has been taken in a bubble chamber, and is schematically represented in Fig. 5. The bubble chamber, as the cloud chamber, constitutes another system to detect elementary particles. Essentially, it consists of a recipient filled with a superheated liquid, i.e., a liquid at a metastable state at a temperature larger than its boiling point. If the liquid is traversed by a charged particle, it will yield the ionization of the liquid molecules along its trajectory and the liquid will vaporize along its path, so that the particle will be made visible as a trace of bubbles. As in a cloud chamber, only charged particles can be detected in a bubble chamber.

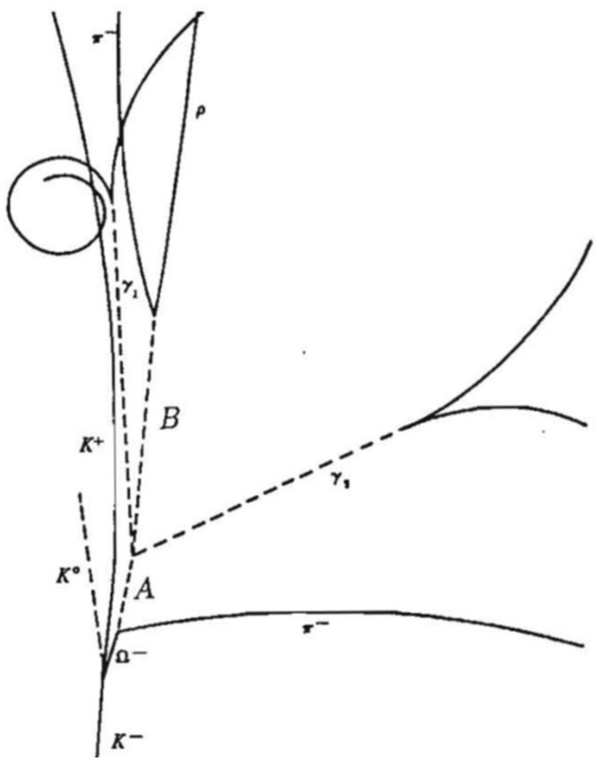

Figure 5:

In the process shown in the figure, a K^- meson collides with a proton at rest. As a result, an Ω^- particle, a K^+ kaon and a neutral K^0 particle, which cannot be seen in the photograph as its is a neutral particle, are created. It is known that the Ω^- particle is disintegrated into a neutral particle A (in principle unknown) and a π^- meson, and that, in turn, the A particle is disintegrated in two photons (γ_1, γ_2) and in another neutral particle B: $A \to \gamma_1 + \gamma_2 + B$. The B particle is detected when disintegrating into a proton and a π^- meson: $B \to \pi^- + p$. The trajectories of the charged particles are curved due to a magnetic field perpendicular to the plane of the figure, $B = 1.70 \pm 0.04$ Teslas.

1. Consider the disintegration process: $B \to \pi^- + p$

   The radii of curvature of the π^- meson and the proton are: $r_{\pi^-} = 50.2 \pm 0.6\,\mathrm{cm}$, $r_p = 2.94 \pm 0.03$ m.

   - Determine the linear momentum of the B particle and the particles resulting from its disintegration (π^-, p).

     To calculate p_B, it is required to determine p_p, p_{π^-} and the angle between the trajectories of the proton and the pion. In order to measure adequately this angle, draw these trajectories on greaseproof paper and measure the angle on this paper. Notice that the trajectories are curved: to determine correctly the angle, draw the tangents to the trajectories at the disintegration point and measure the angle between them.

   - Identify the B particle.

2. Consider now the process: $A \to \gamma_1 + \gamma_2 + B$.

   It is known that the linear momenta of the resulting photons are: $p_{\gamma_1} = 82 \pm 2\,\mathrm{MeV}/c$, $p_{\gamma_2} = 177 \pm 2\,\mathrm{MeV}/c$.

   - Determine the linear momentum of the A particle (in order to do that, it is convenient to project the linear momenta of the resulting particles (γ_1, γ_2, B) along two perpendicular directions X and Y plotted at the disintegration point.

   - Identify the A particle.

Name and symbol		Mass (MeV/c^2)	Charge (units of e)
Positron, electron	e^+, e^-	0.511	± 1
Muon	μ^+, μ^-	105.7	± 1
Pi Meson	π^+, π^-	139.6	± 1
K plus, K minus Mesons	K^+, K^-	493.8	± 1
K zero Meson	K^0	497.8	0
Proton	p^+	938.3	1
Neutron	n	939.6	0
Lambda	Λ^0	1115.4	0
Sigma plus	Σ^+	1189.4	1
Sigma zero	Σ^0	1192.3	0
Sigma minus	Σ^-	1197.2	-1
Xi zero	Ξ^0	1314.3	0
Xi minus	Ξ^-	1320.8	-1
Omega minus	Ω^-	1675.0	-1