Physics Department
Mechanics Laboratory

MOMENT OF INERTIA OF A DISK. THE TORSION PENDULUM.

1. Aim

The aim of this experiment is to determine, through experimental measurements, the moment of inertia of a rigid body and the torsion constant of a helical spring using the dynamic method.

2. Overview

The torsion balance (also called torsion pendulum) is an apparatus devised by physicist Charles-Augustin de Coulomb in 1777 as a means to measure weak forces. It is based upon the resistance that a helical spring opposes to being twisted.

In an elastic spring, the force applied is proportional to the deformation effected on the spring, according to Hooke's Law:

$$F = -k \cdot x \qquad [1]$$

where k is called the elastic constant of the spring (N/m).

There is a similar law for helical springs, except it refers to a moment of force being applied instead of a force, and the deformation is an angular torsion θ:

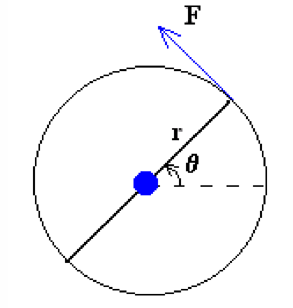

Figure 1

$$\tau = F \cdot r = -K \cdot \theta \qquad [2]$$

K is called the *torsion constant* and it is measured in N·m

Dynamic measurement of the torsion constant

If the supporting disk is separated some angle from its equilibrium position and it let go, the disk begins to oscillate. When the supporting disk is deflected an angle θ and it let go, the spring exerts on the supporting disk a moment of opposite sense to the angular displacement:

$$\tau = -K \cdot \theta \qquad [3]$$

Applying Newton's Second Law to rotational motion:

$$\tau = -K \cdot \theta = I \cdot \frac{d^2\theta}{dt^2} \qquad [4]$$

where I is the total moment of inertia. This expression can be rewritten as:

$$\frac{d^2\theta}{dt^2} + \frac{K}{I} \cdot \theta = 0, \qquad [5]$$

which corresponds to the equation of a simple harmonic motion of frequency $\omega = \sqrt{K/I}$ and period:

$$T = 2\pi\sqrt{\frac{I}{K}} \qquad [6]$$

Moment of inertia of the system

If two identical bodies of known mass are placed symmetrically on the supporting disk at a distance x from the axis of rotation, then the total moment of inertia of the system will be:

$$I = I_D + 2 \cdot I_E \qquad [7]$$

where I_D is the moment of inertia of the supporting disk (including the shaft and tightening bolt) and I_E is the moment of inertia of each mass about the axis of rotation.

The moment of inertia I_E can be calculated by using Steiner's Theorem:

$$I_E = I_C + m \cdot x^2 \qquad [8]$$

where I_C is the moment of inertia about an axis parallel to the axis E that goes through the center of mass, m is the mass of the body, and x is the distance from the center of mass to the axis E.

The moment of inertia of a cylinder of radius R, height h and mass m about the axis of the cylinder is:

$$I_C = \frac{1}{2} m \cdot R^2 \qquad [9]$$

3. Learn more...

• **SERWAY, RA & JEWETT, JW. "FISICA" Volumen 2. 3rd ed. Ed Thomson 2003 Cap. 15 "Movimiento Oscilatorio"**
- 15.5 Péndulo de Torsión

• **ALONSO, M & FINN, EJ "FISICA" Vol. I Ed Addison-Wesley Iberoamericana 1986**
Cap. 12 "Movimiento Oscilatorio"
- 12.6 El péndulo

On the Internet
http://www.sc.ehu.es/sbweb/fisica_/solido/rotacion/torsion/torsion.xhtml

4. Equipment

Figure 2

1. Supporting disk.
2. Two pairs of cylinders of different masses and dimensions.
3. Stopwatch.
4. Ruler and Vernier caliper.

5. Experimental Procedure

5.1 Moment of inertia of the cylinders about the axis of rotation I_E

For each pair of cylinders:

a) Measure their mass on a precision scale and calculate the mean value and the measurement error.

b) Measure their diameter with the Vernier caliper and calculate the mean value and the measurement error. From this measurement, calculate the radius of the cylinders and its error range.

c) Using the ruler, measure the distances X_1, X_2... from the axis of the cylinders to the axis of rotation.

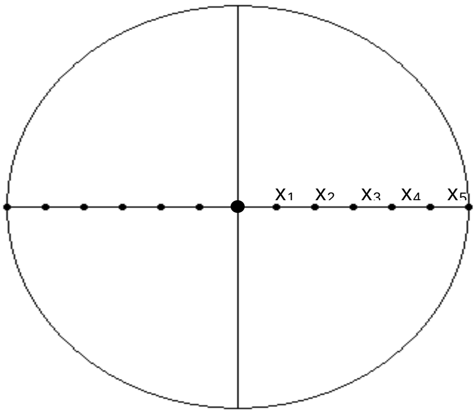

Figure 3

d) For each mass and position X_i of the cylinders relative to the axis E, calculate their moment of inertia I_E with expressions [8] and [9].

5.2 Oscillation period

For each mass and position X_i:

a) Place the pair of cylinders symmetrically on the disk at a distance X_i from the axis of rotation. Turn the disk away from its resting position ($\theta \sim 90º$ -- $180º$) and, using the stopwatch, measure the time it takes the disk to complete five full oscillations (use a lower number of periods if the motion is too damped). Repeat each measurement three times and find the mean value and its error.

b) From this measurement, calculate the period of the oscillations and its error range.

5.3 Torsion constant *K* and moment of inertia I_D of the supporting disk

a) From equations [6] and [7], obtain the expression that links the period squared T^2 to the moment of inertia of the cylinders about the axis I_E.

b) Draw a graph of T^2 versus I_E for each set of mass and position previously measured. Perform a least-squares line fit and draw the fit line.

c) Discuss the meaning of the fit parameters, based on the equation obtained in section a).

d) From the fit parameters, find the value of the torsion constant K of the helical spring and the moment of inertia I of the supporting disk, as well as their respective errors.