HOOKE'S LAW.
THE RATE OR SPRING CONSTANT *k*.

1. Aim.

The aim of this experiment is:

- to study the relationship between the distorting load applied to a spring and the extension (or strain) of this.
- to determine the relationship between a suspended weights on a vertical spring and its period of oscillation.
- to calculate in both situations the spring constant *k.*

2. Overview.

Hooke's law is an approximation of the relationship between the load applied to a body and the deformation that takes place in it. This law is valid for small deformations (within the *elastic limit*), so that when the distorting load is removed the body returns to the original state. In case the load is removed and the body does not return to its original status this deformation is called *plastic.* Both types of deformations occur before the break of the body takes place.

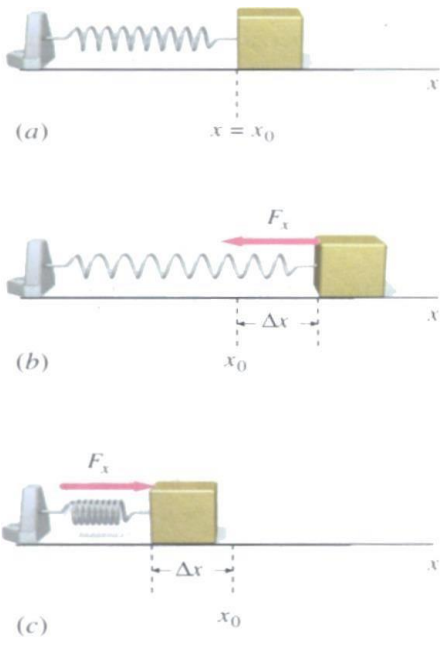


Figure 1

When a body is subjected to an external load $\vec{F}_e$, that produces a deformation $\Delta\vec{x} = \vec{x} - \vec{x}_0$, the body responds by generating an internal, restoring force $\vec{F}_i$ in the same direction and in the opposite sense, so that the sum of the forces will be zero (*body at equilibrium*). If the model of the body to be distorted is a spring and the stress is applied along its axis, the vectorial character of the forces we have mentioned provides their sense (figure 1).

For small deformations, the relationship between stress and strain is linear:

$$F_i = -F_e = -k(x - x_0) = -k\Delta x \qquad \textbf{[1]}$$

where *k* is *the spring constant* (or *rate of the spring*), a measure of the spring's stiffness, which depends on intrinsic properties of the spring such as size and materials it is made of.

Simple harmonic motion.

Equation [1] describes a situation of equilibrium of forces. Suppose now that we stress a spring and then release it. According to Hooke`s law, the spring will want to recover its equilibrium position and the force will be:

$$F_i = -kx \qquad \textbf{[2]}$$

where, for simplicity, we denote by x the position of the spring with respect to its equilibrium position. Since there is not any force that compensates for it, by Newton's second law we have

$$F = ma = m\frac{d^2x(t)}{dt^2} \qquad \textbf{[3]}$$

Setting these two expressions equal to one another gives us the equation

$$m\frac{d^2x(t)}{dt^2} = -kx(t) \qquad \textbf{[4]}$$

where we have expressed the acceleration as the second derivative of the position with respect to time. This equation describes a *harmonic simple motion.* The solution of the equation [4] will give the position as a function of time $x(t)$. A generic function that satisfies equation [4] is:

$$x(t) = A\cos(\omega t + \delta) \qquad \textbf{[5]}$$

where A is the amplitude, ω is the angular (or radian) frequency and δ is the phase. Figure 2 shows a plot of previous solution, where the periodic oscillation of the position can be clearly seen.

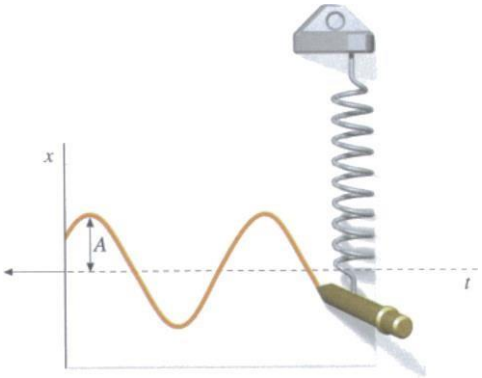


Figure 2

If we substitute the value $x(t)$ given by equation [5] in equation [4] and make operations, we conclude that

$$-Am\omega^2\cos(\omega t + \delta) = -Ak\cos(\omega t + \delta) \qquad \textbf{[6]}$$

The value of both amplitude and phase will be determined by the initial conditions. With respect to the angular frequency, according to expression [6], it should be equal to:

$$m\omega^2 = k \qquad \textbf{[7]}$$

We know that the cosine is periodic, with a period of 2π. If we denote by T the period (time needed to return to the initial position) we obtain that

$$\omega T = 2\pi \qquad \textbf{[8]}$$

Finally, making operations between expressions [7] and [8], we obtain a relationship between the period and the physical constant of the spring:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \textbf{[9]}$$

Such expression characterizes the harmonic oscillator that does not depend on the oscillation initial conditions.

3. To learn more

• TIPLER, PA & MOSCA, G. "Physics" Volume 1. 5-th edition Ed Reverté 2005
Ch 4 "Newton's Laws"
- 4.4 Forces in nature: Springs

Ch 14 "Oscillations"
- 14.1 Simple harmonic motion

On the Internet

http://en.wikipedia.org/wiki/Hooke's_law
http://en.wikipedia.org/wiki/Simple_harmonic_motion
http://www.sc.ehu.es/sbweb/fisica/dinamica/trabajo/muelle/muelle.htm

4. Equipment

1. Springs.

2. Spring support.

3. Graduated ruler.

4. Pointer.

5. Set of weights.

6. Weight holder.

7. Chronometer.

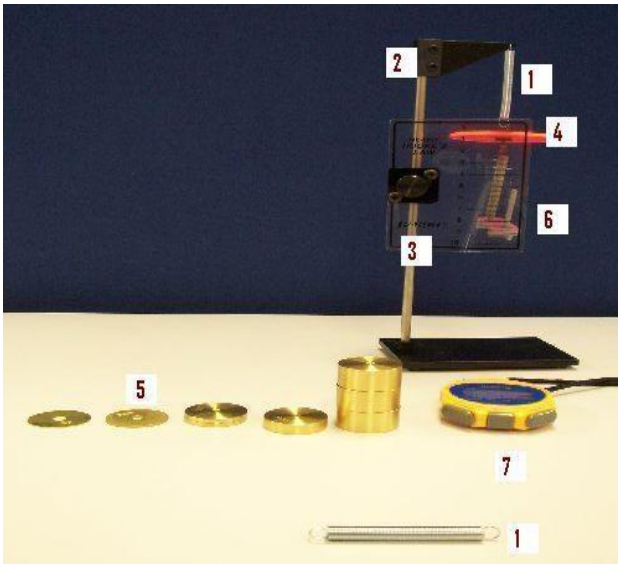


Figure 3

5. Experimental procedure

5.1 Relationship strain-stress.

5.1.1 Experimental setup.

Consider the setup shown in figure 3, for each of the springs. Make sure that the pointer does not hit the graduated ruler and take measurements when the spring stops oscillating. Use for the "zero height" $\mathbf{h_0}$ the length marked by the pointer without weight in the holder.

5.1.2 Data acquisition.

Add carefully the set of weights one by one, and measure the length of the deformed spring. Use the mass values as shown **in the tables provided together with the springs**. Red spring: Use an initial mass of 20g and add 10g at a time. Blue spring: Use an initial mass of 80g and add 20g at a time.

5.2 Relationship period-mass

5.2.1 Experimental setup.

Consider the setup shown in figure 3 without the graduated rule.

5.2.2 Data acquisition.

With the same range of weights as in previous section, slightly stretch the spring with respect to the equilibrium state (no more than 2cm). Measure the time that it takes to complete 25 oscillations. Repeat the procedure 3 times for each weight and spring.