



# COLLISIONS ON AIRTRACK

## 1. Aim

- The aim of this experiment is to illustrate the first two of Newton's Laws of Motion,
- and analyze the conservation of (linear) momentum and kinetic energy in elastic and inelastic collisions.

## 2. Overview

The subject of particle dynamics is the determination of the motion of a particle, given the causes (i.e., forces) that cause this motion. In order to do this, the first step consists on identifying the forces that act upon the particle. The second step involves applying Newton's Laws. Thus, to be able to determine the motion of a particle knowing the forces that act upon it, it is necessary to have good understanding of the underlying principles in Newton's Laws. On the other hand, applying Newton's Laws involves a differential equation which is not always simple. However, conservation laws, which are most often easy to apply, can provide interesting results about motion.

According to Newton's second law:

$$\sum \vec{F} = \frac{d\vec{P}}{dt} \quad [1]$$

where the sum includes all forces acting upon the body, and vector  $\vec{p} = m\vec{v}$  is the linear momentum. Assuming that the body mass remains constant, equation [1] can be expressed as:

$$\sum \vec{F} = m \frac{d\vec{v}}{dt} = m\vec{a} \quad [2]$$

Since mass is always a positive constant,  $\vec{F}$  and  $\vec{a}$  are two vectors with the same direction and sense, so the change in the body's velocity happens in the same direction as the applied resultant force.

### 2.1 Collisions. Conservation of linear momentum.

When two particles or bodies collide, their velocities change. This implies that the bodies exert forces onto each other. The forces that appear in the collision are the internal forces. If the resultant of all external forces happens to be nil, then the system composed of the two bodies is isolated and the total momentum of the system is conserved. The following relationship is met:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f} \quad [3]$$

where subscripts 1 and 2 refer to each of the bodies, and subscripts  $i$  and  $f$  refer to initial and final time instants.

Generally, total kinetic energy is not conserved in the collision. Only in perfectly elastic collision is kinetic energy conserved. In the collision, part of the initial kinetic energy is

transformed into elastic energy, as bodies are deformed. In elastic collisions, both bodies recover their original shape and all of the elastic energy transforms back into kinetic energy. Thus, all initial kinetic energy is recovered, although it can be distributed in a different way after the collision. So, if the collision is perfectly elastic:

$$\frac{p_{1i}^2}{2m_1} + \frac{p_{2i}^2}{2m_2} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2} \quad [4]$$

If particle 2 is initially at rest and the collision is one-dimensional, from equations [3] and [4] we can derive the following:

$$\vec{p}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} \vec{p}_{1i} \quad [5]$$

$$\vec{p}_{2f} = \frac{2m_2}{m_1 + m_2} \vec{p}_{1i} \quad [6]$$

We can point out, from equations [5] and [6], several interesting cases:

- $m_1 = m_2 \rightarrow p_{1f} = 0$  and  $p_{2f} = p_{1i}$ : Particle 1 stops and particle 2 starts moving with the same initial velocity as 1 had before.
- $m_1 \gg m_2 \rightarrow p_{1f} \approx p_{1i}$  and  $p_{2f} \approx 2m_2 v_{1i}$ : Particle 1 hardly loses any velocity, while 2 is shot with twice the velocity of 1.
- $m_1 \ll m_2 \rightarrow -p_{1f} \approx p_{1i}$  and  $p_{2f} \approx 0$ : Particle 1 bounces back with the same velocity it had but in the opposite sense, and 2 does not move perceptibly.

### 3. Learn more...

- **Física, Caps 4, 5 y 7, Paul A. Tipler, 3ª Edición, Ed. Reverté, S.A. (1994).**
- **Física Para Ciencias e Ingenierías, Vol. 1, Raymond A. Serway, 6ª Edición, Ed. Thomson.**
- **Física, Vol. 1, La naturaleza de las cosas, Caps. 4, 5 y 6, Susan M. Lea, John Robert Burke, Internacional Thomson**

### 4. Equipment



Figure 1

1. Air track.
2. Two photogates.
3. Photogate controller.
4. Wiring: A power cord to connect the controller to the power socket, and two connectors to connect the photoelectric sensors to the controller.
5. Two gliders.
6. Glider accessories that allow several kind of measurements to be taken with the photogates.
7. Several devices that allow to accelerate the glider with a string and pulley, additional masses for the gliders, and bumpers for performing elastic and inelastic collisions.
8. Ruler.

## 5. Experimental Procedure

### 5.1 Isolated body.

In this first part we shall study uniform rectilinear motion of a body and verify if friction is negligible in our experimental set-up, which will allow us to consider the glider an *isolated body*.

#### Experimental set-up

- a) Take a glider (5) and attach to it the device (6) that allows to take measurements with the photogates.
- b) Place the glider on the air track (1).
- c) Place the two photogates (2) next to the glider in such a way that the gliders can go under it but the opaque card stops the beam when passing through. The distance between gates should be about 40 cm.
- d) Connect both photogates to the controller (3).
- e) Plug the controller into the power socket.

The experimental set-up should be like the one in Figure 2.



Figure 2

## Measurements

Measure the time that takes for 1 cm of the transparent plaque (Figure 3) through the photogate. The glider's velocity will be  $v = \frac{1}{t}$  cm/s.

To take this measurement, first engage the air pressurizer using the controls under the table (Figure 4), then turn on the power button and turn right the "AIR OUTPUT" switch, up to half the scale. If the air track is level, then the glider should stay motionless.

To determine the time taken by the centimeter on the transparent plaque going through each gate, follow these steps (Figure 5):

1. Turn on the photogate controller.
2. Press button 1, "select measurement". The screen reads "Time".
3. Press button 2, "select mode". The screen reads "Time: One Gate".
4. Press button 3, "start/stop". The screen will show an asterisk, indicating it is ready to measure.
5. Carefully push the glider and write down the reading of the time through the first gate,  $t_1$  (seconds).
6. Before the glider goes through the second gate, press again the button 3, "Start/Stop", to measure the time through the second gate,  $t_2$ .

Repeat this procedure under the following sets of conditions:

- a) Increasing the distance between gates up to 80 cm.
- b) Placing an additional mass on the glider (on both sides of the glider, so that it remains level, Figure 6) and with 40 cm separation between gates.

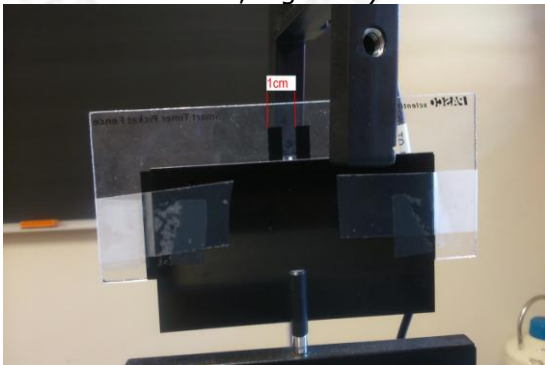


Figure 3

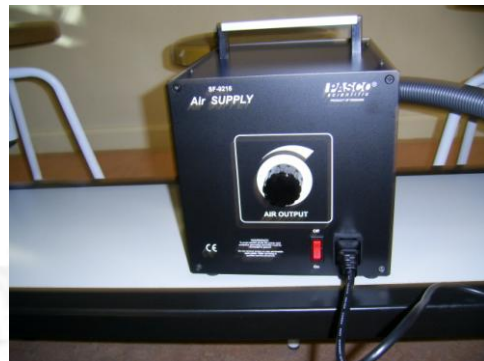


Figure 4



Figure 5

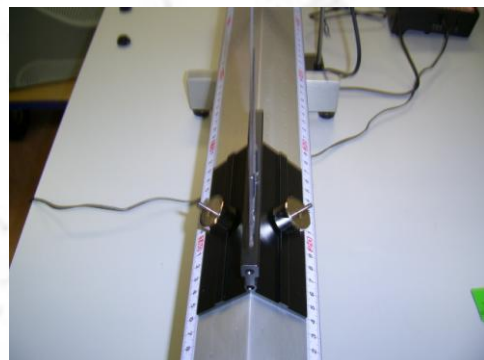


Figure 6

## 5.2 Uniformly accelerated motion.

In this section we will study uniformly accelerated motion and its relationship with Newton's Second Law of Motion.

### Experimental set-up

- a) Take a glider (5) and attach to it the device (6) that allows to take measurements with the photogates.
- b) Attach to this glider the hook device (it is in the box, 7), so it can be pulled with a string. Place it on the air track (1).
- c) Attach the pulley that is in the box (7) to an end of the air track.
- d) Tie one end of the string to the hook on the glider, and hang a weight from the other end (box, 7). Make the string go through the pulley.
- e) Place the two photogates (2) in such a way that the gliders can go under it but the opaque card stops the beam when passing through. The distance between gates should initially be about 40 cm.
- f) Connect both photogates to the controller (3).
- g) Plug the controller into the power socket.
- h) Place the glider 10 cm from the first photogate.

The experimental set-up should be like the one in Figures 7 and 8.



Figure 7

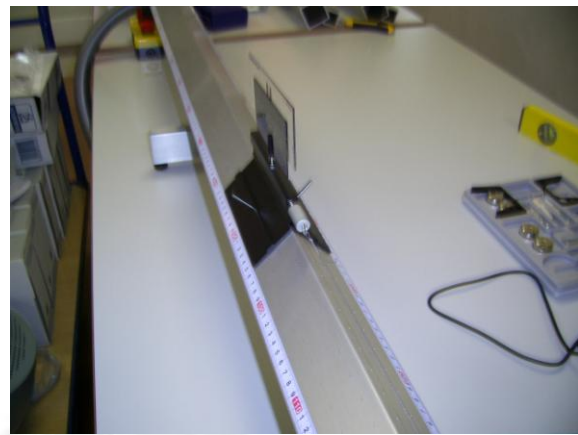


Figure 8

### Measurements

The glider will initially be at rest, and it will be accelerated by the fall of the weight placed at the end of the string. Take two measurements in each experiment: the time taken by the glider in covering the distance  $L$  between the gates, and the time taken by the glider in advancing one centimeter through the photogate. Both measurements cannot be taken at the same time. So, **it is very important to repeat the experiment under the same exact conditions: the glider must be initially at the same place** (for example, 10 cm from the gate), the separation between gates must be the same, and the weight hanging from the string must be the same. Take several measurements for different distances between photogates. Be sure to leave the first gate at the same spot (at 10 cm from the glider) and move only the second gate.

To take the measurements, engage the air pressurizer and follow this procedure:

- 1) Turn on the photogate controller.
- 2) Press button 1, "select measurement". The screen reads "Time".
- 3) Press three times the button 2 "Select Mode" until the screen reads "Time: Two

Gates".

- 4) Press button 3, "start/stop". The screen will show an asterisk, indicating it is ready to measure.
- 5) Release the glider. The controller will give the time  $t_1$  in seconds that the glider has taken in covering the distance  $L$ .
- 6) Place the glider on its initial position. It is necessary that this is the exact same position as when taken the previous measurement.
- 7) Press three times the button 2 "Select Mode" until the screen reads "Time: One Gate".
- 8) Release the glider. After it has gone through the first gate, press button 3, "Start/Stop" before it reaches the second gate. The interval measured,  $t_2$ , is the time that takes the glider to move one centimeter through the second gate.

Measure these intervals for six different distances between gates, from 25 to 75 cm in 10 cm steps.

### Calculations

- Calculate the velocities after going through the second photogate (use interval  $t_2$  as datum) and draw a graph of velocity versus the time interval  $t_1$  that it takes the glider to cover the distance  $L$ .
- Perform a least-squares line fit with these experimental points. Take into account that:

$$v(t) = v_o + at \quad [7]$$

- From Newton's Second Law, and considering that the force applied onto the glider is  $\vec{F} = M\vec{g}$  we can derive that:

$$Mg = (M + m)a \Rightarrow a = \frac{Mg}{M + m} \quad [8]$$

- Weigh the accelerating mass and the glider to calculate the theoretical value of the acceleration of the glider.  $M$  is the accelerating mass, and  $m$  is the mass of the glider and hook. Compare the theoretical value to the experimental value and discuss the results.
- Draw a graph of the separation  $L$  between gates versus the time that takes the glider to cover this distance. What law of physics does this graph represent? Could you perform a line fit of these data to obtain the acceleration in this motion?

## 5.3 Collisions

In this section we will carry out experiments on elastic and inelastic collisions. In elastic collisions, both momentum and kinetic energy are conserved, while in inelastic collisions only momentum is conserved.

### 5.3.1 Inelastic collisions. Experimental set-up

- a) Take both gliders (5) and attach to them the transparent plaque (6) that allows to take measurements with the photogates.
- b) Place on a side a glider a needle attachment (it is in the box, 7), and on the opposed side of the other glider the wax-filled cylinder attachment.
- c) On the free side of both gliders place a similar but flat-ended accessory (see Figures

- 9 and 10) to keep them balanced.
- d) On the end of the air track, where the gliders will bump, place a rubber band bumper.
  - e) Place the two photogates (2) in such a way that the gliders can go under it but the opaque card stops the beam when passing through.
  - f) Connect both photogates to the controller (3).
  - g) Plug the controller into the power socket.
  - h) Place one of the gliders between the photogates and the other one in front of the first gate.

The experimental set-up should be like the one in Figure 12.

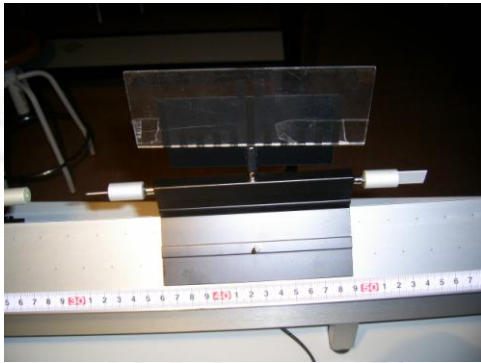


Figure 9

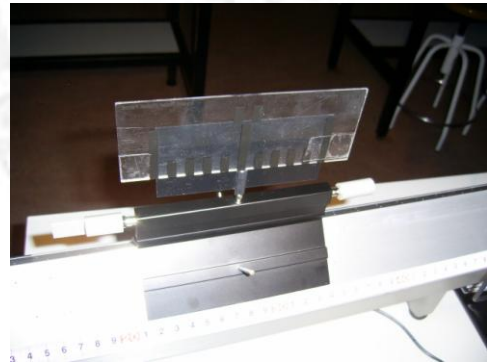


Figure 10

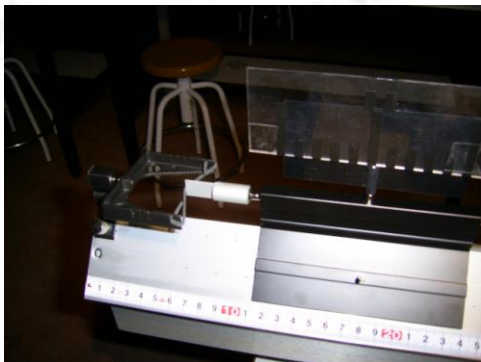


Figure 11



Figure 12

## Measurements

As previously indicated, one of the gliders will be between the photogates and the other will be in front of the first one. Push the glider that is in front of the first gate so that it bumps on to the other glider and both remain stuck together after the collision. Measure the velocity with which the first glider goes through the first photogate and the velocity with which both go through the second gate. It is important that the separation between the two gates is enough to take the measurements, and that the two gliders remain stuck together after the collision.

To take the measurements, engage the air pressurizer and follow this procedure:

- 1) Turn on the photogate controller.
- 2) Press the button 1, "Select measurement" several times until the screen reads "Speed".
- 3) Press three times the button 2 "Select Mode" until the screen reads "Collision". This

option allows to measure the initial velocities of each glider before the collision, and then their velocities after the collision. The units that appear are cm/s.

- 4) Press button 3, "start/stop". The screen will show an asterisk, indicating it is ready to measure.
- 5) Push the first glider into the one that is between the gates. The gliders will collide, bounce back on the elastic bumper at the end of the track and pass again through the gates. At the end of the process, the screen will read: **1 x, y**. This is, the number "1" followed by two numbers that we will refer as **x** and **y**.
- 6) In order to understand what happened, it is necessary to explain how the controller has measured. After pressing the "Start/Stop" button, the controller waited for two passes through gate 1 and two passes through gate 2. When gate 1 measured two passes and gate two another two passes, the controller showed up on the screen: **1 x, y**. The number **1** refers to "gate 1", **x** is the velocity of the first pass through the gate 1 (in cm/s) and **y** is the velocity of the second pass through the gate 1. If we press the button 2, "Select Mode", the screen will read: **2 z, v**. The number **2** refers to "gate 2", **z** is the velocity of the first pass through the gate 2 and **v** is the velocity of the second pass through the gate 2.
- 7) Now we can understand what happened: The first glider was launched and passed through gate 1, and this gate measured the velocity value **x**. After, the glider collided with the second one and both, stick together, approached gate 2. When the first glider passed through gate 2, this gate measured the velocity value **z**. When the second glider passed through gate 2, this gate measured the velocity value **v**. Now both gliders together bounce back on the elastic bumper at the end of the track and the pass again through the second gate (no measure is made) and after they pass through gate 1, this gate measure the velocity value **y**.
- 8) We are interested in measure **x** (initial velocity of the first glider at gate 1) and measure **z** (*velocity of both gliders after the first pass through the second gate*).

Increase the mass of the glider that is initially at rest (the one between both photogates) and repeat the measurement for this new situation.

### 5.3.2 Elastic collisions. Experimental set-up

- a) Take both gliders (5) and attach to them the transparent plaque (6) that allows to take measurements with the photogates.
- b) Attach to an end of one glider a "bumper blade" (flat-ended accessory) and a rubber band bumper to the opposite end of the same glider (both are in the box, 7); see Figure 13.
- c) On the other glider attach the same accessories, but the opposite way. The flat end on one of them will hit the rubber band on the other one (see Figure 14).
- d) Attach a bumper blade and a rubber band bumper to the ends of the air track, so that the gliders can bounce on them.
- e) Place the two photogates (2) in such a way that the gliders can go under it but the opaque card stops the beam when passing through.
- f) Connect both photogates to the controller (3).
- g) Plug the controller into the power socket.
- h) Place one glider between the two photogates.

The experimental set-up should be like the one in Figure 15.



Figure 13

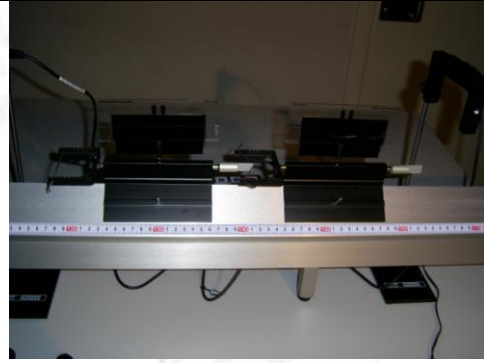


Figure 14



Figure 15

### Measurements

As previously indicated, one of the gliders will be between the photogates and the other will be in front of the first one. Push the glider that is in front of the first gate so that it bumps onto the other glider. Measure the velocity of both gliders before and after the collision, from the values measured when they pass through the photogates. In order to do this, follow the same procedure as in section 5.3.2. The experiment should evolve as follows: the first glider is launched and passes through the first gate with velocity  $x$ , collides with the second glider and stops. The second glider is launched forward and passes through the second gate with velocity  $z$ , bounces back on the elastic bumper at the end of the track and passes again through the second gate with velocity  $v$ . Finally, the second glider collides with the first glider, stops, and the first glider passes through the first gate with velocity  $y$ . You should understand that the  $y$  measure is **not used because it corresponds to a second collision** and not to the one we are measuring.

Increase the mass of the glider that is initially at rest (the one between both photogates) and repeat the measurement for this new situation. You should realize that now the  $y$  measure is used because it gives us the final velocity of the first glider, on its second pass through gate 1 (**of course it is of fundamental importance that this pass happens before the second glider bounces back and collides for second time with the first glider**).