1. Goal.

Calculation of the velocity of light in air and in transparent solid and liquid bodies.

2. Overview

The velocity $v_m$ of propagation of an electromagnetic wave (light) in a material medium, obtained from Maxwell’s equations, is given by:

$$v_m = \frac{1}{\sqrt{\varepsilon_r \cdot \mu_r \cdot \mu_0}} = \frac{1}{\sqrt{\varepsilon \cdot \mu}} \quad [m/s] \quad [1]$$

where:
- $\varepsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$ is the electric field constant (or permittivity of the vacuum).
- $\mu_0 = 4\pi \times 10^{-7} = 1.257 \times 10^{-6} \text{ Hm}^{-1}$ is the magnetic field constant (or permeability of the vacuum).
- $\varepsilon_r$ y $\mu_r$ are the relative permittivity and permeability of the medium, such that $\varepsilon_r = \varepsilon / \varepsilon_0$ and $\mu_r = \mu / \mu_0$, respectively. For the vacuum, $\varepsilon_r = \mu_r = 1 \leftrightarrow \varepsilon = \varepsilon_0$ and $\mu = \mu_0$.

The electric permittivity and magnetic permeability of a material describes how the electric and magnetic fields of an electromagnetic wave interact with it. In the most general case, in an isotropic material the relative permittivity and permeability are complex magnitudes, whose real parts correspond to the electromagnetic energy stored inside the material and whose imaginary parts are related to loss, that is, it is a measure of how the electromagnetic wave’s energy diminishes when this passes through the material.

The refractive index of a medium $n$ is defined as the ratio between the velocity of light in the vacuum $c$ and in the medium $v_m$:

$$n = \frac{c}{v_m} = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = \sqrt{\varepsilon_r \cdot \mu_r} \quad [2]$$

In general, it is a function of the radiation wavelength and the nature of the medium it propagates through, giving rise to light dispersion phenomena in materials. Therefore, we can say that the velocity of propagation of radiation depends on the radiation wavelength. This is the reason why we can observe that a change in the direction of a ray of light passing from one material to another depends on the radiation wavelength, when these
have different refractive indices.

For most transparent media we can consider that $\mu_r = 1$. As we have mentioned, in the vacuum: $\varepsilon_r = 1$ and $\mu_r = 1$; hence:

$$v_m = c = \frac{1}{\sqrt{\varepsilon_r \cdot \varepsilon_0 \cdot \mu_r \cdot \mu_0}} = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = 2.997 \times 10^8 \text{[m/s]}. \ [3]$$


- Optical bench (supporting platform for the optical system, composed of deviating mirrors and lenses).
- Operating unit.
- Lenses on magnetic stands (2).
- Inverting mirror system with screw adjustments.
- Incandescent lamp (12V AC).
- Cylindrical vessel of length $L_c = (100.0 \pm 0.1) \text{ cm}$ with liquid.
- Block of synthetic resin, with longest edge of length $L_B = (30.0 \pm 0.1) \text{ cm}$
- Analog dual-channel oscilloscope with a bandwidth of 35 MHz.
- Coaxial cables.

4. Constants and magnitudes of interest.

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}; \quad \mu_0 = 1.257 \times 10^{-6} \text{ H/m}; \quad C = 2.998 \times 10^8 \text{ m/s}$$

$$v_{H_2O} = 2.248 \times 10^8 \frac{m}{s}; \quad v_{\text{resin block}} = (1.87 \pm 0.01) \times 10^8 \frac{m}{s}; \quad v_{\text{air}} = (2.99 \pm 0.01) \times 10^8 \text{ m/s}$$

$$n_{H_2O; \tau=20^\circ C} = 1.333 \pm 0.001; \quad n_{\text{resin block}} = 1.597 \pm 0.003; \quad n_{\text{air}} = 1.000 \pm 0.001$$

5. Experimental Procedure.

5.1 Description of the Experiment.

![Experimental setup diagram]

Fig. 1 Experimental setup diagram.
The operating unit in figure 1 contains a light source -transmitting diode (red light LED)- whose intensity is modulated at frequency $f_m = 50.1\, MHz$. The LED light passes through a lens (Lens 1), which allows focusing it on its “outbound” way with the inverting mirror system. This makes the light return to the operating unit, but now, with the help of a second lens (Lens 2), it falls upon the photodiode (receiver), which generates an alternating signal of the same frequency $f_m$, but with a phase difference with respect to the signal that modulates the intensity of the transmitting source. This way, a displacement in the inverting mirror system with respect to the plane that contains the transmitter and the receiver in the operating unit will bring along a change in the path traversed by the light between them. Therefore, the phase difference between both signals will also change.

For a displacement $\Delta x$ in the mirror position, there will be a phase difference $\Phi$ between the transmitter and the receiver signals. In this case, the time interval $\Delta t$ in which light traverses the distance $L = 2\Delta x$ between the transmitter and the receiver is given by:

$$\Delta t = \frac{\Phi}{2\pi f_m} \, [s]. [4]$$

We should take into account that the phase of a periodic signal of the form
$$\cos(\omega t + \Phi)$$
is
$$\Phi = \omega \Delta t \leftrightarrow \Delta t = \frac{\Phi}{2\pi f}.$$ 

The velocity $v_L$ of light traversing the distance $L$ is:

$$v_L = \frac{L}{\Delta t} = \frac{2 \Delta x}{\Phi} \frac{4\pi f_m \Delta x}{\Phi} \, [m/s]. [5]$$

From this we conclude that, by measuring the phase difference $\Phi$ between the transmitter and the receiver signals, if we know the increment $\Delta x$ in the path light traverses, we can determine its velocity $v_L$.

In order to carry out an appropriate measurement of the phase difference between two signals of the same frequency, as in our case (remember that the signals from the transmitter and the receiver have the same frequency and differ only in their phase if there are changes in the length of the path the ray traverses). One method for measuring phase shift is to use XY mode, called like this because both the X and Y axis are tracing voltages. The waveform that results from this arrangement is called a Lissajous pattern or figure. We can access this with the X-Y mode of the dual-channel oscilloscope. After pressing the (X-Y) button, if the signals in the input channels X and Y of the oscilloscope correspond to two simple harmonic movements of the form
$$\begin{align*}
x(t) &= A \sin(2\pi f_x t + \phi_x), \\
y(t) &= A \sin(2\pi f_y t + \phi_y),
\end{align*}$$
and these are superposed across perpendicular directions, we obtain a set of Lissajous figures, whose specific shapes depend on the relationship between the frequencies of both signals $[f_x/f_y]$ and their phase difference $[\phi = \phi_x - \phi_y]$. 
In case that input signals have the same frequency \( f_x = f_y \) we observe a set of Lissajous figures, whose shape depends on the phase difference \( \phi \) between signals in both channels, as shown in Figure 2. Once the figure is displayed, we must properly select the sensitivity (volts/div.) the oscilloscope shows the input signals with. An appropriate choice for the sensitivity is that in which the figure spreads over the largest area in the oscilloscope display.

<table>
<thead>
<tr>
<th>( \Phi = \phi_x - \phi_y )</th>
<th>0</th>
<th>( \frac{\pi}{4} )</th>
<th>( \frac{\pi}{2} )</th>
<th>( \frac{3\pi}{4} )</th>
<th>( \pi )</th>
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<tr>
<td>( f_x/f_y = 1 )</td>
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*Fig. 2. Lissajous patterns for signals with the same frequency and phase difference equal to 0; \( \pi/4; \pi/2; 3\pi/4 \) and \( \pi \), respectively.*

Remember that in our experiment, signals are modulated at frequency \( f_m = 50.1 \, MHz \), but this sampling frequency is too high for a conventional oscilloscope, with a bandwidth of 35 MHz, which will show less than a 70% of the signal amplitude at 50.1 MHz. For this reason, the frequency is decreased to 50 kHz by means of a mixer in the service unit.

5.2 Set-up and Optical System Adjustment.

*(ask your laboratory teacher before proceeding with this part)*

The operating unit is set in the extreme without scale of the supporting platform, with the front side (where controls are located) oriented towards the observer (the elevated edge). The lenses are fixed to the magnetic stands and placed in front of the transmitter (LED) and receiver (Photodiode), with the plane sides oriented towards the diodes and parallel to the plane containing the LED and the receiving photodiode (on the lateral side of the operating unit).

The X-output of the operating unit corresponds to an electrical voltage proportional to the modulation voltage of the transmitter, with a frequency of 50.1 kHz, whereas the Y-output corresponds to an electrical voltage proportional to that generated at the photodiode when the light reflected in the mirror system falls upon it. Both outputs are connected to the channels 1 and 2 of the oscilloscope with the BNC sockets, respectively.

**Check that:**

- The X signal in the operating unit connected to channel 1 of the oscilloscope is a sinusoid of frequency 50 kHz.
The plane side of the lenses must be at a distance between 3.5 and 4 cm of the plane containing the LED and the photodiode. At the back of the inverting mirror system we can find an auxiliary incandescent lamp, which helps find the right position for the lenses. The lamp is connected to a 12V AC output in the operating unit. By placing the lamp holder in its natural position on the platform, the wire filament is at the optical axis of either the transmitter or the receiver, depending on the terminal chosen to connect the lamp to its holder. The wires supplying power to the lamp are plugged into the free terminals in the opposite extreme. Once the lamp is on, we must wave the lens slightly on the lamp’s optical path until the focal spot falls upon the LED or the photodiode.

When the lenses have been adjusted, the lamp is switched off by unplugging the wires from the operating unit and the mirror system is rotated so that it stays in the way of the optical path of the ray from the transmitter (that is, a rotation of 180° with respect to the previous position). If the adjustment is correct, we see the signal corresponding to the photodiode on the oscilloscope display (Y-output of the operating unit, connected to the channel 2 of the oscilloscope). Next, we delicately tighten the screws that keep the mirrors parallel. We can maximize the amount of (LED) light that reaches the photodiode by making very small changes in the lenses position; this corresponds to a higher signal amplitude in the channel 2 of the oscilloscope.

Taking into account the difficulties of the adjustment and alignment process of the optical system, in most of the cases you will find that the system has been previously adjusted by the technical staff. Therefore, in case of not being able to measure the signals at the operating unit output, you should NOT change the lenses and mirrors positions without checking first with your laboratory teacher. In general, the reason why signals are not detected at the operating unit is a wrong connection or configuration of the oscilloscope in the X-Y mode.

Check that:

- The inverting mirror system is placed at the zero point of the scale of the supporting platform.
- If the system of lenses and mirrors is correctly adjusted, we see a sinusoid of frequency 50 kHz and amplitude not smaller than 1V at the Y-output of the operating unit connected to channel 2 of the oscilloscope. Otherwise we must readjust the optical alignment of the system.
- The mirror system is slowly slid along the graduated scale of the platform until 150 cm, checking at all times that the signal amplitude in the channel 2 of the oscilloscope, though weakened, does not vanish. In this position, if we observe that the amplitude in channel 2 is smaller than 0.5V, we delicately readjust the mirrors.

5.3. FIRST EXPERIMENT: Measurement of the velocity of light in air.

Once the system is set up and adjusted, we proceed in the following way:

1. Place the inverting mirror system at the zero point on the scale of the supporting platform.
2. Connect the oscilloscope and set the X-Y mode. A Lissajous figure appears on the display. Choose the sensitivity of both channels so that the figure spreads over the screen.
3. With the “Phase” control in the operating unit, we adjust the phase difference between the two signals until a straight line appears on the oscilloscope display, of slope either 1 or -1. This indicates that the phase
difference $\Phi$ between the signals is 0 or $\pi$, respectively (see figure 2). If the position of the mirror system at which the straight line appears does not correspond to the scale zero point, we note the position at which this occurs and designate it by $x_i$.

4. The mirror system is slid along the platform until we observe a straight line with slope opposite to that of the initial one. That is, the change in the length of the path traversed by light from the transmitter to the receiver corresponds to a change of $\pi$ in the phase difference of signals in channels X (transmitter) and Y (receiver) of the oscilloscope. We write this mirror displacement down and denote it by $x_f$.

The mirror position at which this occurs corresponds to a phase difference $\Phi = \pi$ between the transmitter and receiver signals. The path traversed by the light in this new position has changed by an amount $L = 2\Delta x = (x_f - x_i)$. By making use of equation [5] we can determine the velocity value in air as:

$$v_{air} = \frac{L}{\Delta t} = \frac{4\pi f_m \Delta x}{\Phi} = 4f_m(x_f - x_i) \text{ [m/s].} \ [6]$$

Steps 1 to 4 must be repeated five times to obtain an average value $\bar{L} = \frac{L}{5}$ which allows minimising the random error that comes implicit with the measurement process to determine the velocity of light in air.

### 5.4. SECOND EXPERIMENT: Measurement of the velocity of light in a transparent solid.

In this case, we compare the velocity of light in the liquid or the solid block to that in air $v_{air}$ (previous part). We proceed in the following way:

1. Place the inverting mirror system at position $x_i = 120 \text{ cm}$ (measurement one with the block; see figure 3).
2. Place the resin block somewhere, either in the ray’s outbound or return path between the transmitter (LED) and the receiver (photodiode), with the longest edge, of length $L_B$, parallel to the optical path, so that the light passes through it.

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**Fig. 3** Experimental diagram for the measurement of the velocity of light in a solid or liquid medium.
3. Check that a Lissajous figure appears on the oscilloscope display. With the "Phase" control in the operating unit we adjust the phase difference between the two signals until a (Lissajous) straight line of slope either 1 or -1 appears on the oscilloscope display. This indicates that the phase difference $\Phi$ between the signals is 0 or $\pi$, respectively.

4. Remove the block from the platform to clear the optical path of the light.

5. Slide the mirror system towards the end of the platform until the Lissajous figure shown on the oscilloscope becomes a straight line with the same slope as that of the initial one, when the resin block was in the optical path of the ray. This new position of the mirror system is denoted by $x_f$ (measurement two, without the block; see figure 3).

Since the phase difference between the transmitter and the receiver signals has been kept constant (same Lissajous figure) we can state that light has spent the same amount of time $t_1$, in traversing both distances.

In the first measurement light has passed through a distance $L_1 = 2x_i$. As light crosses the block through its longest edge, of size $L_B$, the total amount of time, $t_1$, is the sum of the time spent by light going through the block, $t_B$, plus the time going through air, $t_{air}$:

$$ t_1 = t_B + t_{air} = \frac{L_B}{v_B} + \frac{(L_1 - L_B)}{v_{air}} \quad [s] \quad [7] $$

where:

- $t_B; v_B$: are the time it takes light to traverse the block and the corresponding velocity.
- $t_{air}; v_{air}$: are the time it takes light to travel outside the block through air and the corresponding velocity.

In the second measurement, the distance light traverses is:

$$ L_2 = L_1 + 2\Delta x = L_1 + 2(x_f - x_i) \quad [m]. $$

Since the time spent in both cases is the same, we have:

$$ t_1 = \frac{L_2}{v_{air}} = \frac{L_1 + 2(x_f - x_i)}{v_{air}} \quad [s]. \quad [8] $$

By setting this equal to the time obtained in equation [7], we get:

$$ \frac{L_B}{v_B} + \frac{(L_1 - L_B)}{v_{air}} = \frac{L_1 + 2(x_f - x_i)}{v_{air}} \quad [s] $$

$$ \frac{L_B}{v_B} - \frac{L_B}{v_{air}} = \frac{2(x_f - x_i)}{v_{air}} \quad [s]. \quad [9] $$

From equation [9], since we know from previous part the velocity of light in air, $v_A = c_L$, we can calculate the velocity of light in the solid block $v_B$ as:

$$ v_B = \frac{L_B v_{air}}{L_B + 2(x_f - x_i)} \quad [m/s]. \quad [10] $$
5.5. THIRD EXPERIMENT. Computation of the refractive index of a liquid.

In this case, we make use of the average velocity of light in air, $v_{\text{air}}$, obtained in the first part to obtain the refractive index of the liquid filling a cylindrical vessel, placed in the path traversed by light between the transmitter and the receiver. We proceed in the following way:

1. Fill the cylindrical vessel with the liquid whose refractive index we want to determine. Place it on its holder, parallel to the platform plane, either in the ray’s outbound or return path between the transmitter (LED) and the receiver (photodiode). The light must pass through the transparent optical windows in the extremes of the cylindrical vessel.

2. Place the inverting mirror system next to the cylindrical vessel, between 5 and 10 cm away from the end closest to the vessel. This position of the mirror system is denoted by $x_i$ (measurement one, with the vessel; see figure 3).

3. Check that a Lissajous figure appears on the oscilloscope display. It corresponds to a certain phase difference between the input signals. With the “Phase” control in the operating unit we adjust the phase difference between the two signals until a (Lissajous) straight line of slope either 1 or -1 appears on the oscilloscope display. This indicates that the phase difference $\Phi$ between the signals is 0 or $\pi$, respectively.

4. Remove the cylindrical vessel with the liquid from the platform to clear the optical path of the light.

5. Slide the mirror system towards the end of the platform until the Lissajous figure shown on the oscilloscope becomes a straight line with the same slope as that of the initial one. This new position of the mirror system is denoted by $x_f$ (measurement two, without the cylindrical vessel; see figure 3).

Since the phase difference between the transmitter and the receiver signals has been kept constant (same Lissajous figure) we can state that light has spent the same amount of time $t_1$ in traversing both distances. By working in a similar manner to that in the measurement of velocity of light in the resin block, we obtain an expression equivalent to equation [9] for the distance in the liquid, $L_L$:

$$\frac{L_L}{v_L} - \frac{L_L}{v_{\text{air}}} = \frac{2(x_f - x_i)}{v_{\text{air}}} \quad [s]. \quad [11]$$

Bearing in mind the definition of refractive index and by solving in equation [11], we get:

$$n_L = \frac{v_{\text{air}}}{v_L} = 1 + \frac{2(x_f - x_i)}{L_L}, \quad [12]$$

where:

- $v_L$: velocity of light in the liquid.
- $L_L = 100.0 \pm 0.1 \text{ cm}$: length of the light path in the liquid.
- $v_{\text{air}}$: velocity of light in air.

In the exercises to calculate the velocity of light in the solid and the refractive index of the liquid, distances $x_i$ and $x_f$ must be measured five times in order to minimise the random error.

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