Department of Physics
Electromagnetism and Optics Laboratory

MAGNETIC MOMENT IN A MAGNETIC FIELD

1. Goal of the experiment

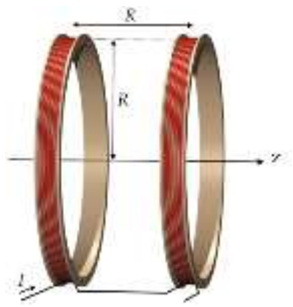
Characterization of the magnetic field created by two current-carrying coils connected as a Helmholtz arrangement.

Measurement of the torque exerted on a magnetic moment in a uniform magnetic field.

2. Overview

Magnetic field created by a Helmholtz arrangement

Two identical circular coils of radius R, each with N turns, that are placed symmetrically along a common axis and separated by a distance equal to R are called Helmholtz coils. The coils carry equal steady current I in the same direction. The main feature of Helmholtz coils is that the resultant magnetic field between the coils is very uniform.



The magnetic field $\vec{B}$ created by a circuit can be calculated by using the Biot-Savart law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \oint_{circuit} \frac{I\,\overrightarrow{dl} \times (\vec{r} - \overrightarrow{r'})}{\left|\vec{r} - \overrightarrow{r'}\right|^3} \qquad [1]$$

where $\overrightarrow{r'}$ locates the current element $I\overrightarrow{dl}$ and $\vec{r}$ locates the point where the magnetic field is calculated. Consider the Helmholtz arrangement axis as the z-axis of a cylindrical (r,z,φ) coordinate reference system, The origin of this reference system is at the center of the arrangement. The exact calculation of the magnetic field is mathematically complex, but some interesting points can be highlighted:

a) The magnetic field is rotationally symmetrical about the axis of the coils. Then, the magnetic field created by the Helmholtz coils does not depends on the φ-coordinate. Only axial and radial components must be taken into account

$$\vec{B} = \overrightarrow{B_z}(z,r) + \overrightarrow{B_r}(z,r) \qquad [2]$$

b) The mathematical problem simplifies when the calculation is restricted to points along the symmetry axis (z-axis). In this case

$$B_z\,(z) = \frac{\mu_0\, I\, N}{2\,R}\left[\frac{1}{(1+A_1^2)^{3/2}} + \frac{1}{(1+A_2^2)^{3/2}}\right] \qquad [3]$$

$$A_1 = \frac{z + R/2}{R} \qquad\qquad A_1 = \frac{z - R/2}{R}$$

where N is the number of turns in each coil, and I is the current carried by the coils.

c) The magnetic field at the center of the arrangement (z=0) is

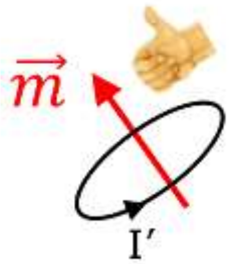
$$B_z(0) = \frac{\mu_0\, N}{R}\left(\frac{4}{5}\right)^{3/2} I = K_H\, I \qquad [4]$$

K_H is a constant and it is called the calibration factor for the arrangement.

Torque on a current carrying loop in a uniform magnetic field

When a closed current loop is placed in a magnetic field, not only a magnetic force but also a torque is exerted on the loop. For a loop of N' turns carrying a current I', the magnetic moment is defined as

$$\vec{m} = N' I' \vec{A} \qquad [5]$$



The vector $\vec{A}$ has a magnitude equal to the area of the loop, and has a direction that is perpendicular to the plane of the loop and determined by using the right-hand rule.

The torque exerted on the loop by a uniform magnetic field is

$$\vec{\tau} = \vec{m} \times \vec{B} \qquad [6]$$

The magnitude of this vector is

$$\tau = m\, B \sin\alpha = N'\, I'\, A\, B \sin\alpha \qquad [7]$$

being α the angle between $\vec{B}$ and $\vec{A}$

As it has been showed above, two coils in a Helmholtz arrangement is a magnetic device that generates a very uniform magnetic field in a wide region inside the arrangement. If the current loop is located at the center of the Helmholtz coils, the magnitude of the torque can be written as

$$\tau = N' K_H\, I\, I' A \sin\alpha \qquad [8]$$

This torque tends to twist the current loop so that its plane would be perpendicular to the direction of the magnetic field. The dc motor or the attitude control (adjustment of their orientation) in many satellites use this twist of coils in magnetic fields.

3. Learn more…

Magnetic field lines of a Helmholtz coil
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/helmholtz.html

Helmholtz and Maxwell coils
https://www.scielo.br/scielo.php?pid=S1806-11172020000100486&script=sci_arttext

The dc motor
https://nationalmaglab.org/education/magnet-academy/watch-play/interactive/dc-motor

Attitude control in satellites by magnetorquers
https://www.youtube.com/watch?v=r2Ep3aZ630U

4. Equipment

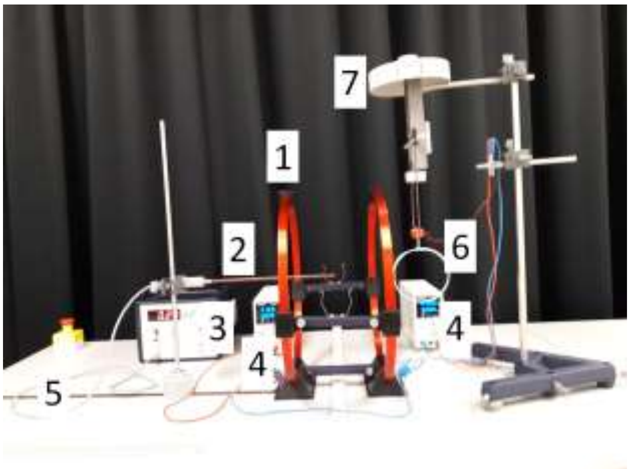


Figure 1: Equipment

1. Helmholtz coils.

2. Teslameter (axial Hall probe).

3. Control unit of the teslameter

4. Power supply.

5. Ruler

6. Circular current loop (3 turns, radius 6 cm).

7. Torsion dynamometer.

5. Set-up and experimental procedure

5.1 Characterization of the magnetic field generated by the Helmholtz coils.

5.1.1 Connecting the Helmholtz coils arrangement.

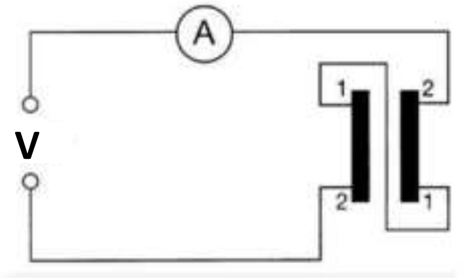


Figure 2: Wiring diagram for the Helmholtz coils arrangement

Connect the two coils of the arrangement in series. The current **MUST NOT EXCEED** 3.5 A

Consider the arrangement axis as the z-axis of a cylindrical (r,z,φ) coordinate reference system, The origin of this reference system is at the center of the arrangement (see figure 3). Taking into account this coordinate system, the magnetic field created by the Helmholtz coils does not depends on the φ-coordinate, and only the components $\overrightarrow{B_r}(r,z)$ and $\overrightarrow{B_z}(r,z)$ have to be measured.

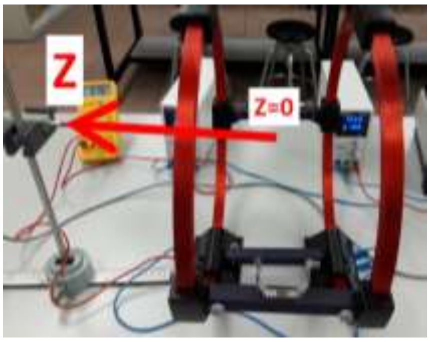


Figure 3: Definition of z-axis and origin for the arrangement

5.1.2 Calibration of the teslameter

- Be sure that the power supply is switched off and no current is circulating by the Helmholtz coils.
- Locate the tip of the teslameter in the place where you want to measure.
- Switch on the control unit of the teslameter. Choose the most sensitive scale position (20 mT).
- Select the "Direct Field" option by using the switch located at the center of the control unit.
- Wait around 5-10 minutes to stabilize the measurement.

Figure 4

- After this stabilization time, the measurement must be 0 mT. If not, use the zero adjustement knob on the control unit to assure that the probe measures 0 mT.
- This teslameter is an axial Hall probe and only measures the magnetic field component in the direction of the probe system. Note that a negative value only means that the vector B is opposite to the one corresponding to a positive value.
- If you can not fit the zero point, try to obtain the lowest value of B, take this value B_{zero} as an offset and correct all your measurements for this offset.

5.1.3 Measuring B_z at the center of the arrangement (z=0, r=0)

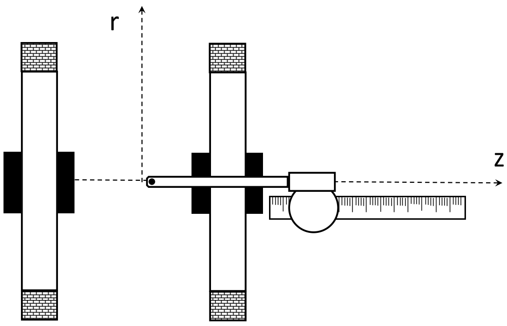


Figure 5: Measuring B_z at the center of the Helmholtz arrangement

- Align the teslameter in such a way that the teslameter axis would be coincident with the direction of z-axis. In this configuration the probe measures the B_z component (see figure 5).

- Locate the tip of the teslameter at the center of the system. Calibrate the teslameter according to the procedure explained at section 5.1.2

- To establish the current in the main circuit follow this procedure
  - a) Turn fully counterclockwise the Current and Voltage knobs.
  - b) Turn fully clockwise the Voltage knob.
  - c) Switch on the power supply.
  - d) Carefully turn clockwise the Current knob until the current measured at the ammeter is 1 A. The current **MUST NOT EXCEED** 3.5 A.

- Measure B_z (do no forget to annotate units and uncertainties)

- Increase the value of the current in steps of 0.5 A until I=3 A. Measure B_z for each step

5.1.4 Measuring $B_z(z)$ along the axis of the arrangement (r=0)

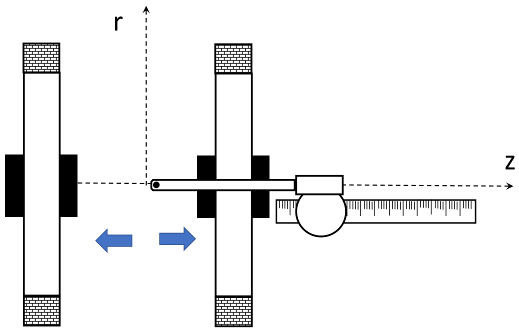


Figure 6: Measuring B_z along the axis of the Helmholtz arrangement

- Align the tip of the teslameter in such a way that it can be moved along the z- axis of the system (see figure 6). In this configuration the probe measures the B_z component.

- Locate the tip of the teslameter at the center of the system. Calibrate the teslameter according to the procedure explained at section 5.1.2

- To establish the current in the main circuit follow this procedure
    a) Turn fully counterclockwise the Current and Voltage knobs.
    b) Turn fully clockwise the Voltage knob.
    c) Switch on the power supply.
    d) Carefully turn clockwise the Current knob until the current measured at the ammeter is 3 A.

- Measure the B_z component along the z-axis (r=0) from the center of the system, in steps of 1 centimeter. Take 20 measurements (some of them will be performed out ot the coil arrangement). Be sure that you have placed and fixed properly the ruler in such a way that the tip moves along the axis.

- Go back to the center of the system and take measurements in steps of 1 centimeter towards the opposite coil until the tip reaches the center of this opposite coil.

5.1.5 Measuring $B_z(r)$ along the perpendicular axis (z=0)

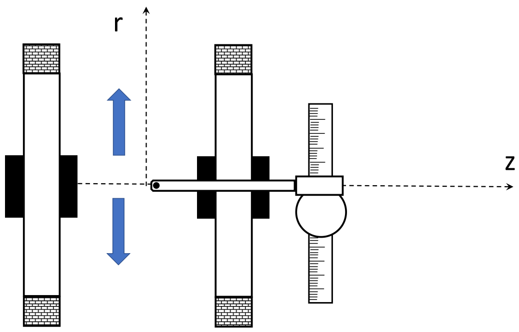


Figure 7: Measuring B_z along the r-axis the Helmholtz arrangement

- Align the tip of the teslameter in such a way that it can be moved along the r-axis of the system (see figure 7). In this configuration the probe measures the B_z component.

- Locate the tip of the teslameter at the center of the system. Calibrate the teslameter according to the procedure explained at section 5.1.2

- To establish the current in the main circuit follow this procedure
  a) Turn fully counterclockwise the Current and Voltage knobs.
  b) Turn fully clockwise the Voltage knob.
  c) Switch on the power supply.
  d) Carefully turn clockwise the Current knob until the current measured at the ammeter is 3 A. .

- Measure the B_z component along the r-axis (z=0) from the center of the system, in steps of 1 centimeter until the probe reaches the end of the coil. Be sure that you have placed and fixed properly the ruler in such a way that the tip moves along the axis and the probe axis remains parallel to the z- axis.

- Go back to the center of the system and then take measurements in steps of 1 centimeter towards the other end of the coil.

5.1.6 Results and questions

- Present all the numerical data in tables, including the units and the uncertainties of the measurements.

- Plot the numerical data measured in 5.1.3 B_z (z=0) vs I .

- Perform a linear fitting of data and calculate the calibration factor K_H from the slope of the fitting (see equation [4]). Include the determination of its uncertainty.

- Plot the numerical data measured in 5.1.4 B_z vs z
  Plot the numerical data measured in 5.1.5 B_z vs r

- From both plots estimate the region between the coils where you can consider B as uniform.

5.2 Determination of the magnetic moment of a current loop in a uniform magnetic field.

5.2.1 Connecting the current loop.

- Place the torsion balance with the current loop in such a way that the plane determined by the loop contains the z-axis of the Helmholtz arrangement. The center of the loop has to be located at the center of the Helmholtz arrangement.

- The connection wires to the loop carrier should hang loosely. They should be twisted together in order to avoid additional torque in the balance.

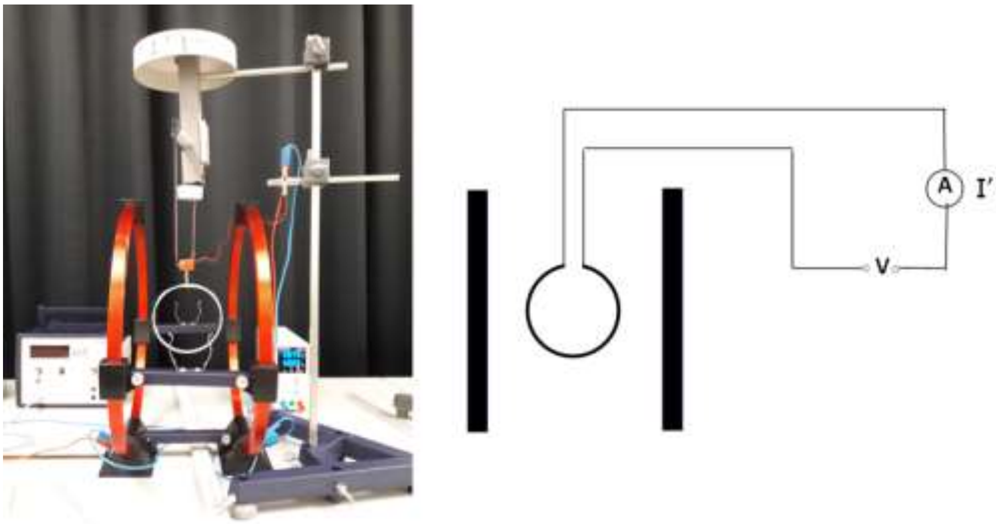


Figure 9: (left) Experimental set-up. (right) Wiring diagram for the current loop

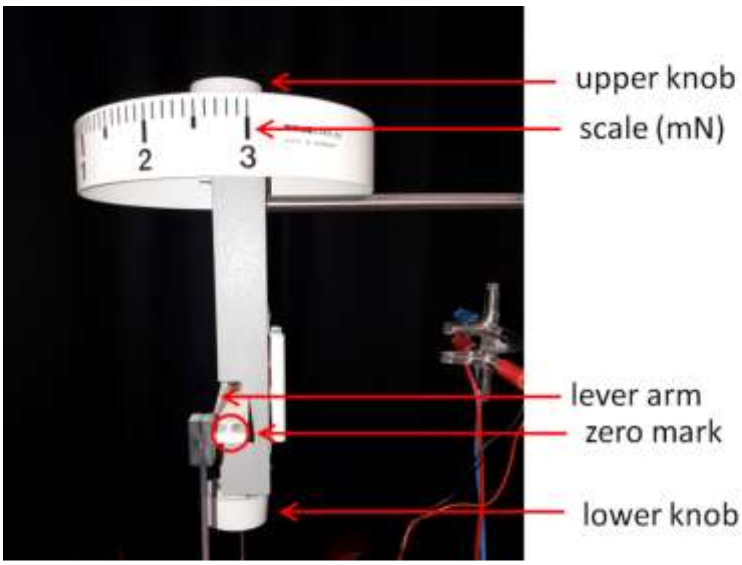


Figure 10: Description of the torsion dynamometer

5.2.2 Calibration of the torsion balance.

- Switch off the power supplies.

- Use the upper knob of the balance to set this force indication knob to zero (see figure 11 left)

- Use carefully the lower knob of the balance to align the lever arm in the middle of the zero indication mark (see figure 11 right).

Figure 11: (left) Calibration of the zero force. (right) Lever arm aligned in the middle of the zero mark

5.2.3 Measurement of the torque due to the magnetic moment in a uniform magnetic field.

- Switch on the power supplies and establish a current I= 1 A in the Helmholtz coils and I'= 5 A in the loop.

- Due to the torque on the loop the balance arm will deflect. Use the upper knob to compensate this deflection until the arm would be aligned again in the zero incication mark.

- Read the force value in the lateral scale.

- The torque can be calculated as

$$\tau = F\,d \qquad [9]$$

where F is the force measured by the torsion balance and d is the diameter of the current loop

- Repeat this procedure by increasing the current in the Helmholtz coils from 1 A to 3 A in steps of 0.5 A. Before each measurement you have to assure that the loop is correctly plced in the Helmholtz arrangement (according 5.2.1) and calibrate the balance (according 5.2.2)

5.2.4 Results and questions

- Present all the numerical data in tables, including the units and the uncertainties of the measurements.

- Plot the torque τ vs the current I carried by the Helmholtz coils

- Perform a linear fitting of data and calculate the magnetic moment of the current loop from the slope of the fitting (see equation [8]). Include the determination of its uncertainty.

- Compare the experimental value of m with the theoretical one $m = N I' A$