Physics Department
Electricity and Magnetism Laboratory

MAGNETIC FIELD CREATED BY SIMPLE CIRCUITS

1. Goal

The aim of this experiment is to calculate the magnetic fields produced by some common current configurations, such as a current loop or a solenoid.

2. Overview

In 1819, the Danish scientist Hans Christian Oerested discovered that an electric current passing through a wire can deflect a magnetized compass needle, proving that electric currents are sources of the magnetic field (just as magnets are)

The magnetic field $\vec{B}$ (*) produced by a current element $I \cdot d\vec{\ell}$ is given by equation [1], known as the Biot –Savart law.

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint_{circ} \frac{d\vec{\ell} \times \vec{r}}{r^3} \qquad [1]$$

Where μ_0 is a constant called the magnetic permeability of free space, having a value:

$$\mu_0 = 4\pi \times 10^{-7} \ \ N A^{-2} \qquad [2]$$

The Tesla (T) is the unit of B in the International System of Units.

- **$\vec{B}$ at the center of a Current Loop.**

Figure 1 shows a current loop of radius R carrying a current I. The magnetic field at the center of the loop (x=0) is directed along the axis of the loop, and its magnitude can be derived from **[1]**.

The magnetic field due to the entire current is found by integrating over all the current elements in the loop. Since R is the same for all elements, we obtain:

$$\vec{B} = \frac{\mu_0 \cdot I}{4\pi \cdot R^2} \oint dl \qquad [3]$$

(*) The correct name for the vector $\vec{B}$ is the *magnetic induction vector*, but in order to simplify we refer to it as the magnetic field vector.

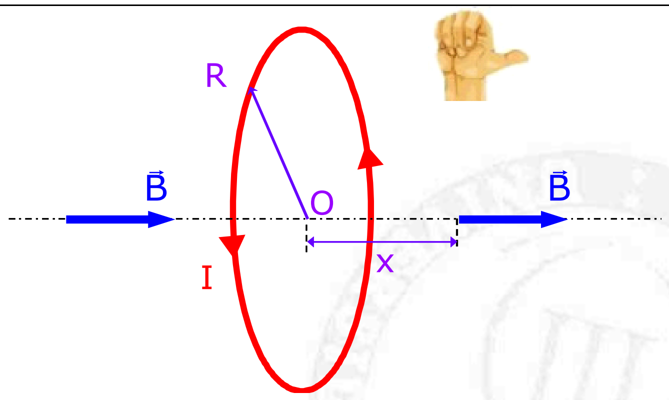


Figure 1

The integral of $d\vec{\ell}$ along the complete loop gives the total length 2πR, the circumference of the loop. The magnetic field due to the entire loop is thus **[4].**

$$B(O) = \frac{\mu_0 I}{2R} \qquad [4]$$

The magnetic field at the center of a loop that has N turns (i.e., a coil) is N times that due to a single turn:

$$B(O) = \frac{\mu_0 N I}{2R} \qquad [5]$$

- $\vec{B}$ **due to a Solenoid.**

A solenoid is a conducting wire tightly wound into a helix of closely spaced turns, as illustrated in Figure 2. The magnetic field of a solenoid is essentially that of a set of N identical current loops placed side by side. Inside the solenoid and far from the ends, the field lines are approximately parallel to the axis and are closely and uniformly spaced, indicating a strong, uniform magnetic field. Outside the solenoid, the lines are much less dense. They diverge from one end and converge at the other end. In figure 3, you can see the magnetic field lines due to a solenoid with a current I (see Fig.3 (a)) and the value of the B field on the axis of the solenoid (see Fig.3 (b)) as a function of distance x to the midpoint of the solenoid.

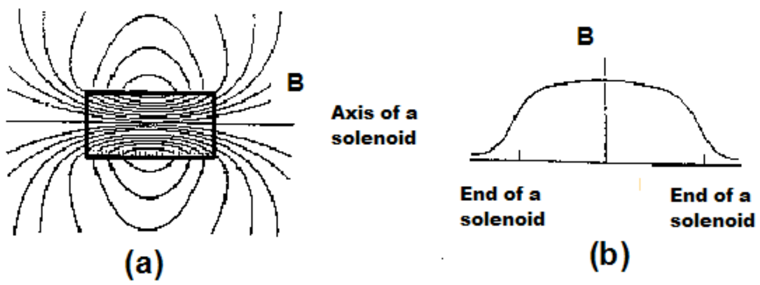
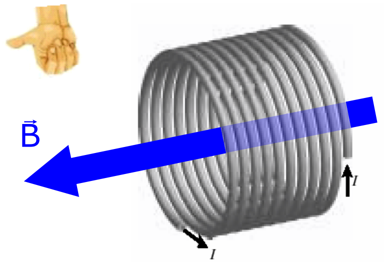


Figure 3



Consider a solenoid of length L consisting of N turns of wire carrying a current I. We choose the axis of the solenoid to be along the *x* axis. The magnetic field due to the entire solenoid can be calculated by integrating expression [1]. The magnitude of B is then given by expression [6].

Figure 2

$$B(x) = \frac{\mu_0 \, I \, N}{2\,\ell}\left(\frac{a}{\sqrt{R^2 + a^2}} + \frac{b}{\sqrt{R^2 + b^2}}\right) \qquad \textbf{[6]}$$

Where

$$a = \frac{\ell}{2} + x$$

$$b = \frac{\ell}{2} - x \qquad \textbf{[7]}$$

The direction of B (along the axis of the solenoid) would be obtained applying the right hand rule.

For a long solenoid for which ***a*** and ***b*** are much larger than R, each of the two terms in parentheses tend to 1. In this case, the magnetic field is.

$$B(x) = \frac{\mu_0 \, I \, N}{\ell}$$

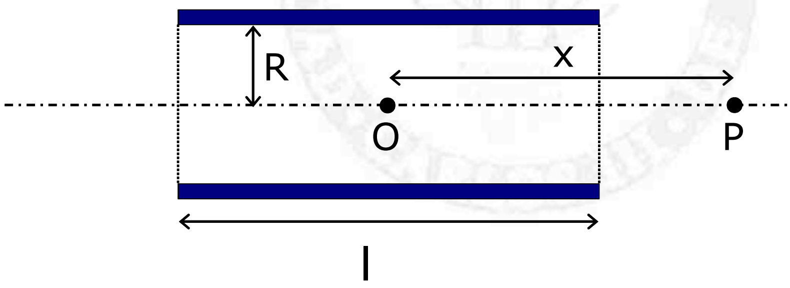


Figure 3

3. Learn more...

• **SERWAY, RA & JEWETT, JW. "Physics" Volume 2. 3th edition Ed Thomson 2003 Chapter 22 "Sources of the Magnetic Field"**
- 22.7 The Biot-Savart Law

• **TIPLER, PA & MOSCA, G. "Physics" 4th edition Ed W.H Freeman and company 1999. Chapter 29 "Sources of the Magnetic Field"**
- 29.2 The Magnetic Field of Currents: The Biot-Savart Law.

In internet:
http://electron9.phys.utk.edu/phys136d/modules/m7/Ampere.htm (en inglés)
http://www.sc.ehu.es/sbweb/fisica/elecmagnet/campo_magnetico/espira/espira.html
http://www.sc.ehu.es/sbweb/fisica/elecmagnet/magnetico/cMagnetico.html
http://www.answers.com/topic/hans-christian-rsted#ixzz1Fu3HPSua
http://www.phywe.com/461/pid/4690/Teslameter,-digital.htm

4. Equipment.

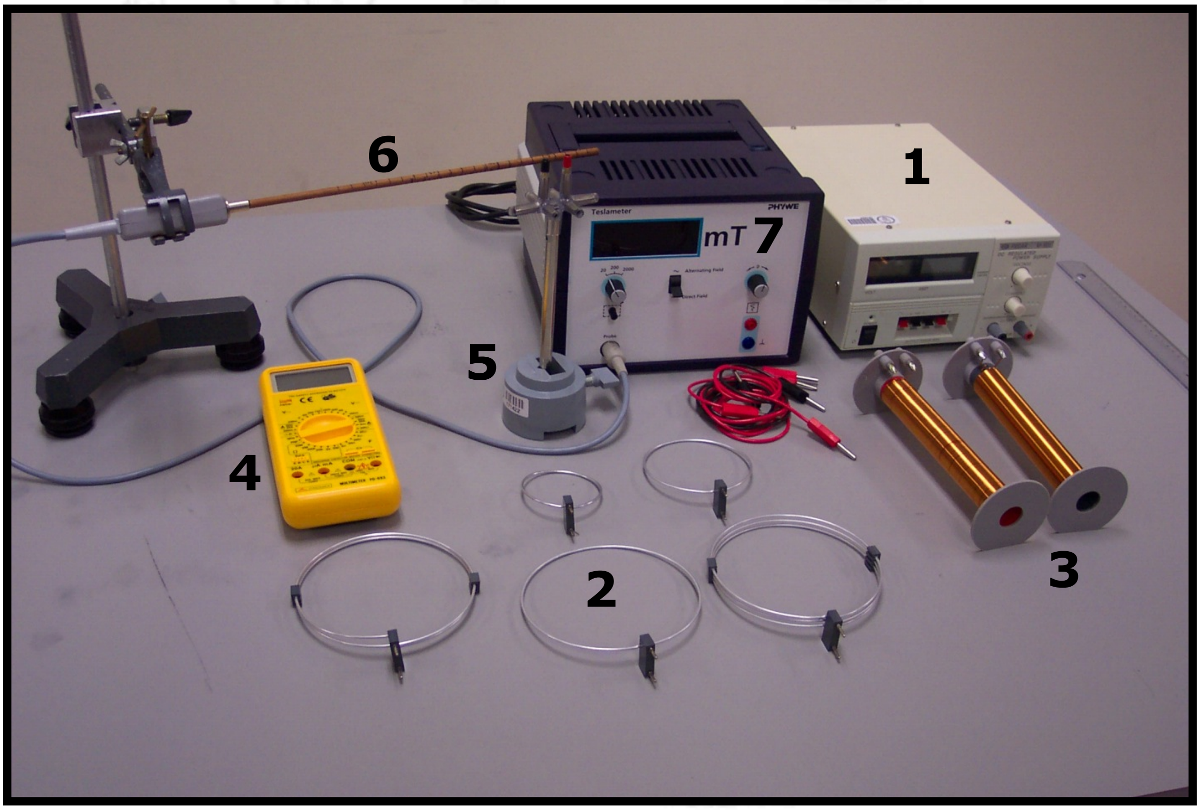


Figure 4

1. Power Supply.

2. Set of current loops.

3. Solenoids.

4. Multimeter.

5. Mounting adapter.

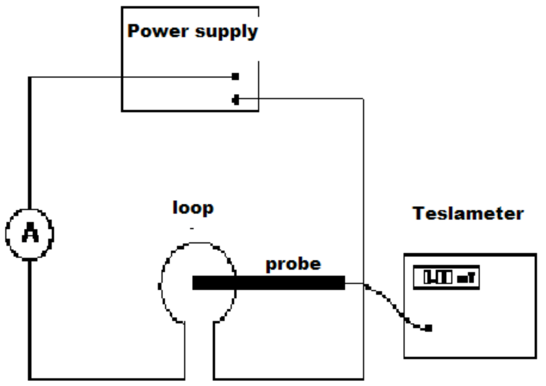
6. Teslameter: Probe.

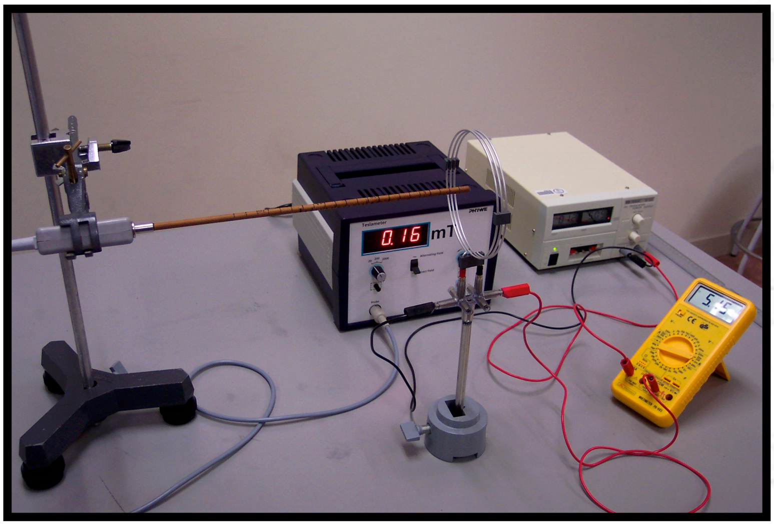
7. Teslameter: Control equipment.

5. Experimental procedure.

5.1 Measurement of the magnetic field at the center of a loop.

5.1.1 General Assembly.





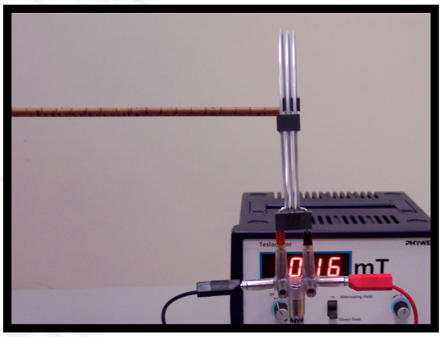


Figure 5: Experimental set-up for measuring a magnetic field.

a) The power supply [1] must be initially off.

b) Place the loop of radius 6 cm and 3 turns, on the mounting adapter.

c) Connect the loop to the power supply. In the circuit connect the multimeter (in ammeter mode) in series to measure the current flowing through the loop. Use the 20A connection.

d) Place the end of the probe at the center of the loop (see figure 5).

5.1.2 Calibration of the teslameter.

Once the general assembly is done, the next step is to calibrate the teslameter. We will turn on the teslameter control equipment, placing the scale in the most sensitive position (20 mT). Check that the switch at the center is set to *Direct Field*.

Before making any adjustment it is advisable to wait ten minutes in order to stabilize the measurement. With the probe placed in the measurement position (center of the loop) and with <u>the power supply off</u> (current flowing through the coil equal to 0), the measure should be 0 mT.

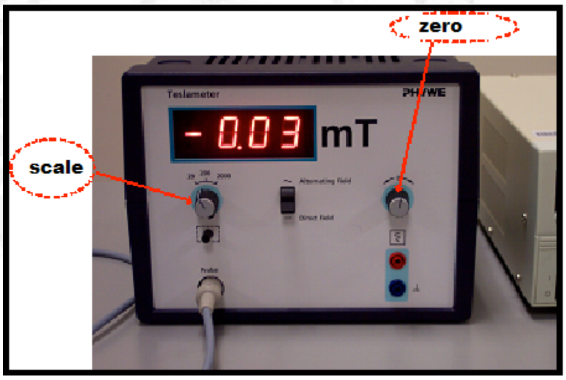


Figure 6

Otherwise, use the zero adjustment wheel of the control equipment to ensure that the meter reads zero. Note that the negative sign that may appear in the panel of the teslameter is only related to the direction of the field $\vec{B}$: a negative value means only that the vector $\vec{B}$ is opposite to the vector associated with a positive value (see Figure 1). If you cannot adjust the value to zero, take the smallest value obtained as the value of B_0. All B values measured in the following paragraphs should be corrected using this "zero" value.

5.1.3 Measurement of the magnetic field at the center of a loop.

Once the zero adjustment is done, we will proceed to measure the B field at the center of the loop.

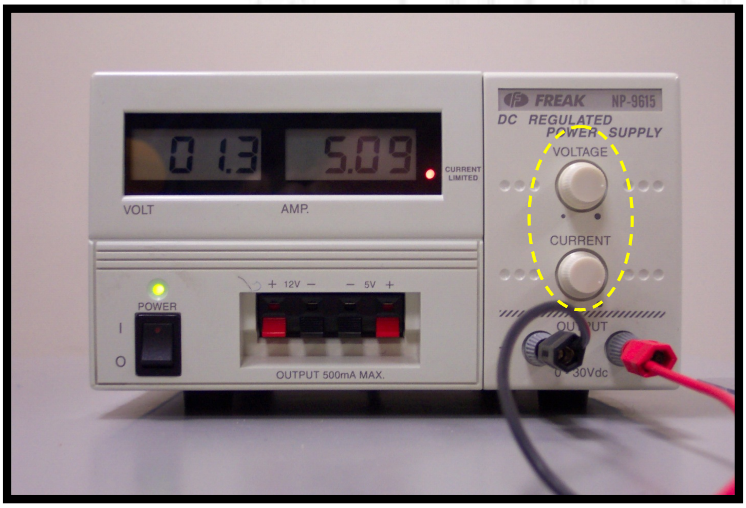


Figure 7

In order to do this:

a) Check that the current (AMP.) and voltage (VOLT.) controls on the power supply (figure 7) is completely turned counterclockwise.

b) Turn the Voltage knob fully counterclockwise.

c) Turn on the power supply.

d) Carefully, turn the Current knob clockwise until the current flowing through the coil is I = 5 A (measure the current using the ammeter, not the power supply).
e) Write down the measurement of the teslameter for the magnetic field B [mT]. Always check that the scale at the teslameter control equipment is located in most sensitive scale. Call this value $\mathbf{B_1}$. Annotate its uncertainty. If the measured B_0 in 5.1.2 is not zero, correct the measurement by obtaining $\mathbf{B_{c1} = B_1 - B_0}$

To eliminate the effects of asymmetry in the experimental setup, repeat the measurement of B by changing the direction the current in the loop. In order to do so
a) Turn the Current knob fully counterclockwise (I = 0 in the circuit)

b) Exchange the terminals of the power supply

c) Carefully turn the Current control clockwise until the ammeter measures the same value for the current.

d) Annotate the measurement for the magnetic field B and the corresponding uncertainty. Call this measurement $\mathbf{B_2}$. If B_0 in 5.1.2 is not zero, correct the measurement by calculating $\mathbf{B_{c2} = B_2 - B_0}$.

Remember that the sign of the value read in the teslameter only indicates the direction of the vector.

Finally, the final value of B is the average of the two measured values in the above procedure:

$$B = \frac{|B_1 - B_0| + |B_2 - B_0|}{2}$$

$$\Delta B = \frac{\Delta B_1 + \Delta B_2 + 2\Delta B_0}{2} \qquad [8]$$

5.1.4 Study of the dependence of B with the number of loops.

The experimental procedure to follow for the study proposed in this section is:

1. Select the coils of radius 6 cm which have 1, 2 and 3 turns.

2. Measure the **B** field at the center of each of these three loops, using the method described in section 5.1.3 and equation [8].

3. Plot B versus the number of turns **N**. Perform a least squares fit and represent the line of best fit.

4. Interpret the meaning of the fitting parameters, according to equation [5].

5.1.5 Study the dependence of B with the radius of the turns.

For the study proposed in this section, select the coils of 1 turn and radii 3, 4.2 and 6 cm. Measure the B field at the center of each of these three loops, using the method described in section 5.1.3 and equation [8].
The experimental procedure to follow is:

1. Select the coils with 1 turn and radios 3, 4.2 and 6 cm.

2. Measure the **B** field at the center of each of these three loops, using the method described in Section 5.1.3 and equation [8].

3. Plot **Ln(B)**versus **Ln(R)**. Perform a least squares fit and represent the line of best fit.

4. Interpret the meaning of the fitting parameters, according to equation [5].

5.1.6 Study the dependence of B with the current.

For the study proposed in this section, the experimental procedure to follow is:

1. Select the coils with 3 turns and radius 6 cm.

2. Measure the **B** field at the center of the loop for different values of current flowing through it. Vary the current between 1 and 5 A in steps of 1 Amp.

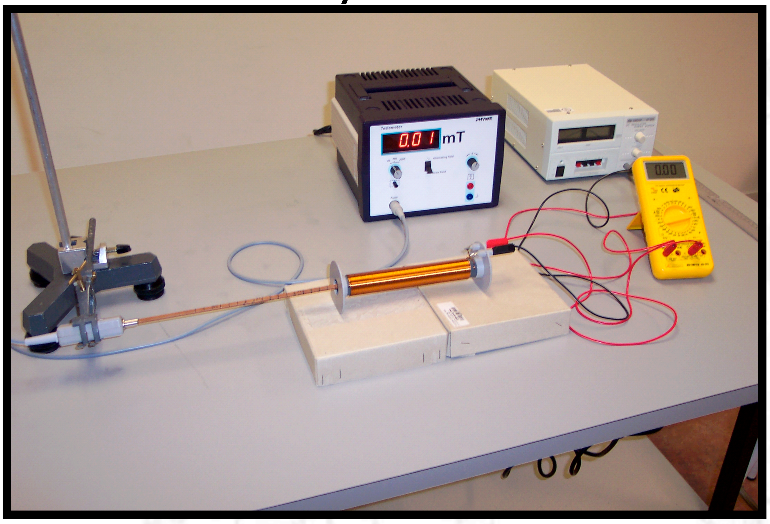
3. Plot the magnetic field **B** versus the current **I**. Perform a least squares fit and represent the line of best fit.

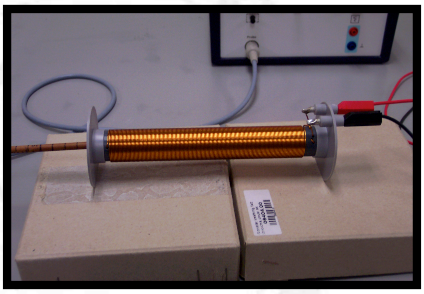
4. Interpret the meaning of the fitting parameters, according to the equation [5].

5. Determine the value of the magnetic permeability of free space μ_0 using the interpretation made in the previous step. Compare the result with the theoretical value given in equation [2].

5.2 Measurement of the magnetic field along the axis of a solenoid

5.2.1 General Assembly. Solenoid with 150 turns.





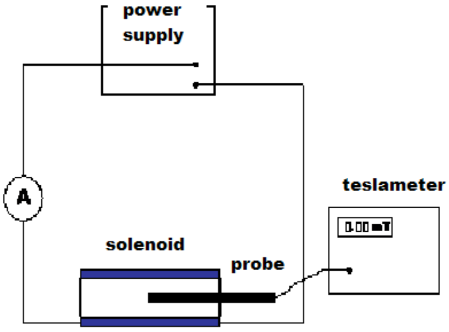


Figure 8: Experimental set-up

1. Start with the power supply and teslameter off, dismount the adapter with the loop and place the solenoid of length 16 cm, 150 turns and radius 1.3 cm

2. Adjust the height of the teslameter probe and place it inside the solenoid at the position until its end is located at the center of the solenoid (use a graduated ruler to determine the position of the center of the solenoid). According to Figure 3, x = 0 corresponds to the center of the solenoid.

3. Adjust $\mathbf{B_0}$ in this position, according to the procedure described in 5.1.2.

4. Regulate the current passing through the solenoid, until 1 A circulates along the circuit.

5. Measure magnetic field B at the position x = 0, using the method described in section 5.1.3. In this case, there is no need to take the measurement of $\mathbf{B_2}$. <u>It is enough to take for each position a single value of **B.**</u>

6. Next, vary the position *x* of the probe end by extracting it in steps of 1 cm until x = 15 cm. Measure the B field at each position of the probe.

7. Place the probe at the center of the solenoid again. Vary the position *x* of the probe end by inserting it in steps of 1 cm, until x = - 15 cm. Measure the B field at each position of the probe.

NOTE: In this case, there is no need to take the measurement of $\mathbf{B_2}$, and it is sufficient to take for each position a single value of $\mathbf{B_1}$.

5.2.2 Solenoid with 75 turns.

Repeat the same steps as above using the solenoid of length 16 cm, 75 turns and radius 1.3 cm.

5.2.3 Questions.

1. Plot the experimental **B** against x for the two solenoids (from -15cm to +15cm).
2. Include in the plot above the theoretical values obtained from equation [6] for both solenoids and positions x = -12, -8, -4, 0, 4, 8 and 12 cm.