INTERFERENCE AND DIFFRACTION

1. Goals

1) To study the diffraction pattern of a single slit,
2) to measure the diameter of a red blood cell,
3) to study the interference between light beams emerging from two closely-spaced slits.

2. Overview

Light is an electromagnetic wave, and under the proper circumstances, it exhibits wave phenomena, such as constructive and destructive interference. The wavelength of visible light ranges from about 400-750 nm, and this wavelength λ sets the scale for the appearance of wave-like effects. For instance, if a broad beam of light partly passes through a wide slit (i.e. a slit which is very large compared to λ), then the wave effects are negligible, the light acts like a ray, and the slit casts a geometrical shadow. However, if the slit is small enough (i.e. around the same size as λ or smaller), then, the wave properties of light become apparent and a diffraction pattern is projected.

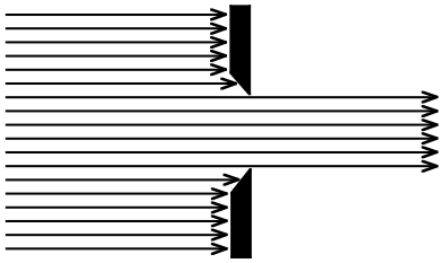


Figure 1a. Slit large compared to λ.

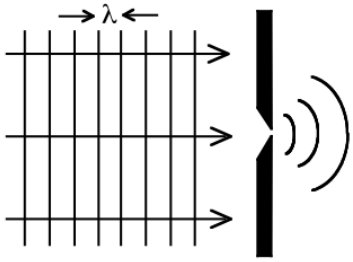


Figure 1b. Slit small compared to λ.

Now consider the light from two coherent light sources a distance d apart. Coherent sources emit light waves that are in phase, or in sync. If we think of light like a water wave, we can imagine that coherent sources emit an identical succession of wave crests and troughs, with both emitting crests at the same time. One way to create such coherent sources is to illuminate a pair of narrow slits with a distant light source.

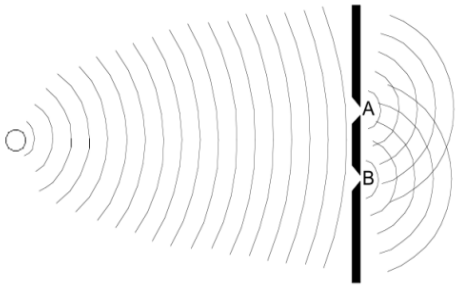


Figure 2: Points A and B acts as coherent sources

Interference from two slits: Consider the light rays from the two coherent point sources made from slits a distance *d* apart (see fig. 3). We assume that the sources are emitting monochromatic (single wavelength) light of wavelength λ. The rays are emitted in all forward directions, but let's concentrate on the rays that are emitted in a direction θ toward a distant screen (θ measured from the normal to the screen, diagram below). One of these rays has further to travel to reach the screen, and the path difference is given by $d \cdot \sin\theta$. When this path difference is exactly one wavelength λ (or any integer number of wavelengths) the interference will be constructive, while when the path difference is $\lambda/2$, $3\lambda/2$, $5\lambda/2$… the interference will be destructive.

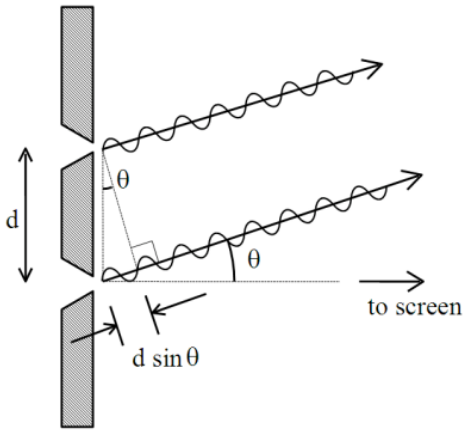
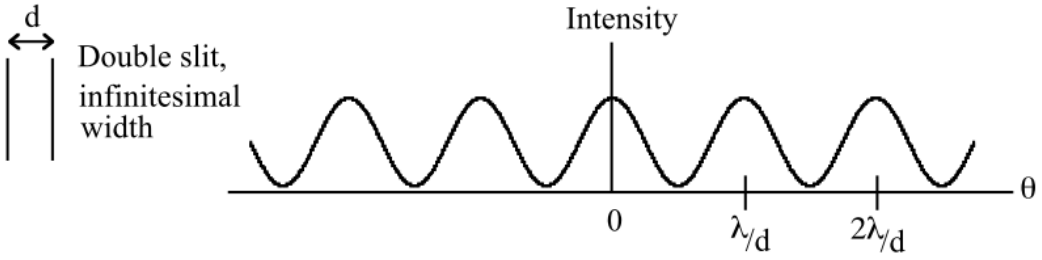


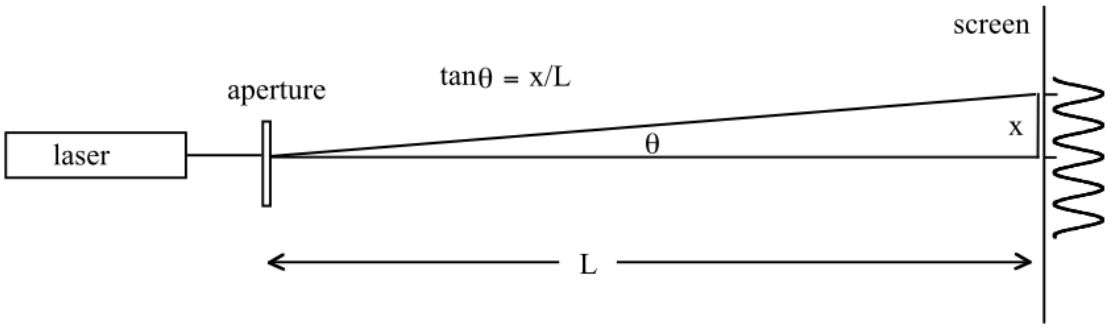
Figure 3.

A complete analysis yields a pattern of intensity vs. angle that looks like:



$$\left.\begin{array}{l} \textit{Bright}: \; d\sin\theta = m\lambda \\ \textit{Dark}: \quad d\sin\theta = (m+\tfrac{1}{2})\lambda. \end{array}\right\} \; \mathrm{m} = 0, \pm 1, \pm 2..$$

Geometric simplification: If θ is small, then $\sin\theta \cong \theta$ (in radians), and maxima occur on the screen at $\theta = m\,\lambda/d$; minima occur at $\theta = \left(m+\frac{1}{2}\right)\lambda/d$. As shown below, the angle θ (measured from the center of the screen) is related to the distance x measured on the screen by $\tan\theta = x/L$, where *L* is the distance from the screen to the source of light (the aperture).



If the angle θ is small (less than a few degrees), then to an excellent approximation, $\sin\theta \approx \tan\theta \approx \theta$ (in radians) so the locations of the interference maxima are given by

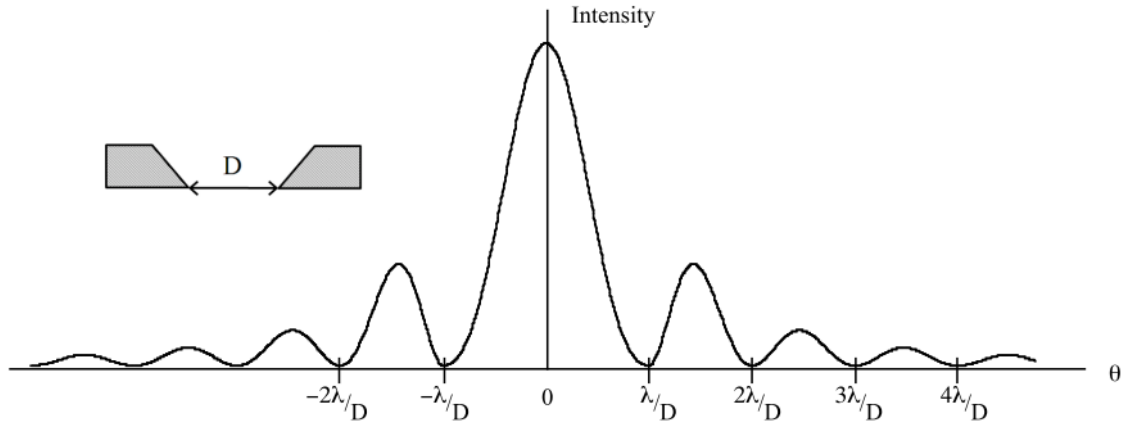
$$\frac{x}{L} = m\frac{\lambda}{d}$$

Single slit diffraction: The uniform 2-slit interference pattern shown above is seldom observed in practice, because real slits always have finite width (not an infinitesimal width). We now ask: what is the intensity pattern from a single slit of finite width *D*? Huygens' Principle states that the light coming from an aperture is the same as the light that would come from a collection of coherent point sources filling the space of the aperture. It´s like if we construct a large slit out of a whole set of small slits, all adjacent to each other. To see what pattern the entire array produces, consider first just two of these imaginary sources: one at the edge of the slit and one in the center. These two sources are separated by a distance *D*/2.

The path difference for the rays from these two sources, going to the screen at an angle θ, is $\frac{D}{2}\sin\theta$, and these rays will interfere destructively if $\frac{D}{2}\sin\theta = \frac{\lambda}{2}$. But the same can be said for every pair of sources separated by *D*/2. Consequently, the rays from all the sources filling the aperture cancel in pairs, producing zero intensity on the screen when $\frac{D}{2}\sin\theta = \frac{\lambda}{2}$ or, if θ is small,

$$\theta = \frac{\lambda}{D} \qquad \text{(First minimum in single slit pattern.)}$$

The complete intensity pattern, called a diffraction pattern, looks like this



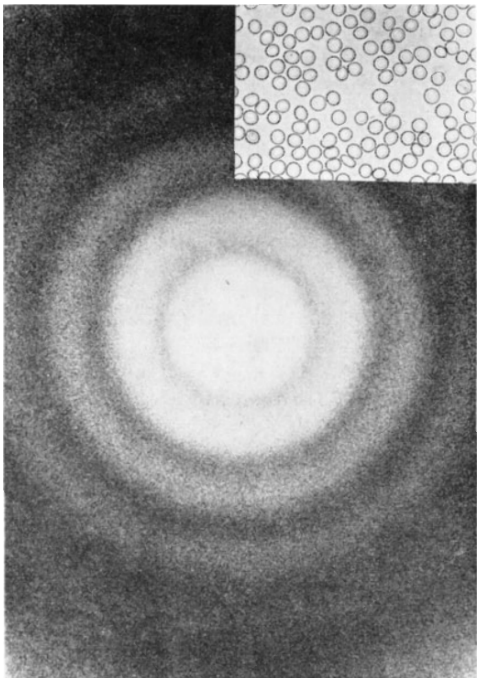
The single slit diffraction pattern has minima at

$$\theta = \pm m\frac{\lambda}{D} \text{ with } m = 1,2,3\ldots \qquad \text{(minima of single slit pattern.)}$$

So, the separation of minima is λ/D, except for the first minima on either side of the central maximum, which are separated by $2\lambda/D$. If *x* is the distance on the screen between minima, then $\theta = x/L = \lambda/D$.

Circular hole diffraction: For a circular hole, interference occurs between all rays from the whole area producing a series of concentric fringes. The exact solution in this case is difficult but turns out to be of the same form as for the single slit ($d\cdot\sin\theta = m\lambda$) except that *m* is related to the zeros of the Bessel function and are given by *m* = 1.22, 2.23, 3.24, 4.24, 5.24 … and *d* is the diameter of the hole.

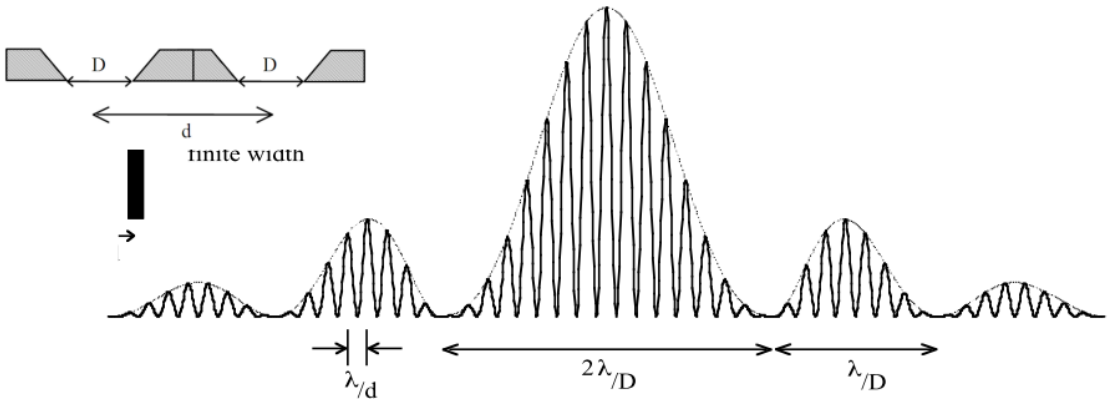
Babinet's principle, which applies to any point outside the area illuminated by the un-diffracted beam, states that the illumination is unaltered if the transparent parts of an aperture become opaque and the opaque parts transparent. Thus, the same pattern of fringes will be produced by a circular hole in an opaque screen and by a circular disk of the same diameter as the hole. The effect of a not too dense and random array of holes is to enhance the intensity of the diffraction pattern due to a single hole. The interference fringes produced by the diffraction of laser light by a random array of holes is showed in the figure (the inset shows the array).



A blood smear on a microscope slide, acts as an array of this type, where particles are red blood cells and diffraction can be used to measure their diameters *d* in the small angular deviation limit through $\sin\theta \approx \theta \approx S_m/L = m\,\lambda/d$, where S_m is the radius of the m^{th} dark fringe, *L* the distance between the diffracting particles and the screen, *m* = 1.22, 2.23m …and λ the observation wavelength.

Combine interference (2-slit) with diffraction (finite-width slit): When the aperture consists of two finite slits, each of width *D*, separated by a distance d, then the intensity pattern is a combination of both the single-slit pattern and the double slit pattern: the amplitude of the two-slit interference pattern is modulated by a single slit diffraction pattern:

In this full pattern, the finely spaced interference maxima are spaced $\Delta\theta = \lambda/d$ apart, while the more widely spaced minima of the single-slit diffraction pattern are separated by $\Delta\theta = \lambda/D$ or $2\,\lambda/D$.



3. Learn more…

- **SERWAY, RA & JEWETT, JW. "Physics" Volume 1.** 3th edition Ed Thomson 2003 Chapters 15,16
- **HALLIDAY D, RESNICK R, WALKER J.** Fundamentals of Physics Vol 2. John Wiley & Sons. Chapters 35,36
- **BOWLT C. "Measurement of red blood cell diameters using a laser".** 1971 Phys. Educ. 6 13. http://iopscience.iop.org/0031-9120/6/1/003.

4. Equipment.

1. Optical table
2. Laser He-Ne (λ=632.8 nm)
3. Mirror
4. Single-slit plate
5. A blood smear on a microscope slide
6. Double-slit plate

5. Experimental procedure.

NEVER LOOK INTO A LASER BEAM.

5.1 Diffraction pattern from a single slit.

The light source will be the He-Ne laser which produces a monochromatic beam with a wavelength of λ = 632.8 nm and a beam diameter of about 1 mm. The power output of our lasers is about 1 mW, a small amount, but still enough to damage your retina if you look directly into the beam.

- Mount the laser so that the beam reflects on the mirror and deflects towards the laboratory wall.
- Mount the single slit near the mirror on the optical table.
- Choose a slit opening.
- Tape a piece of paper to the screen (laboratory wall). You should use this paper to mark the various diffraction and interference patterns that you observe on the screen.
- Adjust the direction of the mirror, and the position of the slit to give the clearest possible diffraction pattern on the screen.
- Measure the distance L.
- Record the slit width *a*.
- Mark on the paper the locations of each of the maxima, making sure to accurately record their positions and widths.
- Remove the paper from the screen.
- Measure the width of the central maximum (the distance between the first minima on either side of the maximum) as marked on the paper. You and your partner should do this twice each.
- Measure the distance between two neighboring dark fringes. In order to increase the accuracy of your measurement, select a group of five neighboring secondary maxima, measure the distance between the minima on either side, and divide it by five. (Can you explain why this procedure improves the accuracy of your measurement?) Again, you and your partner should each do this twice. (Note: Make sure that the central minimum is not part of your group of five.)
- Determine the best possible value for the separation of two dark fringes.
- Compare the width of the central maximum to the average separation of two dark fringes. Is the width of the central maximum twice the separation of two dark fringes, as theory predicts?

- Remount the paper on the screen.
- Change the slit opening and repeat steps 4–12.

1. How does the observed single slit pattern change by varying the width of the slit?

2. For both cases: Obtain experimentally the width of the slit (with the corresponding errors) and compare with the manufacturer´s value.

3. Which measurement is more accurate? Why?

5.2 Diffraction from a red blood cell

- Set up the blood smear on a microscope slide such that the laser light through it.
- Measure the distance *L*.
- Measure the diameter of the first black circle to 5 different directions.

1. Obtain experimentally the diameter of a red blood cell with the corresponding error.

5.3 Double Slit Interference and Diffraction patterns

- Set up the double slit at a distance from the mirror such that both slits are illuminated by the laser light. Select one pair of slits
- Record both the slit widths and the distance between the two slits.
- Measure five groups of 10 bright fringes to obtain a good average value for the distance between two dark fringes in the pattern.
- Repeat steps 1 – 4 using a different double slit.

Note that the fringe separation depends only on *d*, the separation of the two slits. The intensities in the pattern, however, vary with the width of the individual slits. This is because the single-slit diffraction pattern serves as an intensity envelope for the double-slit interference pattern.

1. Compare the diffraction patterns of different double slits to observe that the above rules are in fact correct.

2. Cover one of the two slits and see how the pattern on the screen changes.

3. What happens to the double-slit pattern a) as the distance between the slits is increased; b) as the width of each individual slit is increased?

4. Explain the contributions of both single-slit diffraction and double-slit interference to the double-slit pattern. How should the width of the single slits be chosen to perturb the double-slit pattern as little as possible?

5. For both cases: Obtain experimentally the separation of the slits (with the corresponding errors) and compare with the manufacturer´s value.